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What can low energy quantum systems teach us about space and time?

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Dedicated to
Nonno Uccio

Abstract

Quantum gravity is generally seen as a high energy physics field. However, recent progress in quantum control of matter means we might be able to perform tests of quantum gravity predictions using slow-moving masses. Interest in these kinds of proposals has renewed a conversation between the quantum gravity, quantum information, and quantum foundations communities. This thesis presents original work on topics at the intersection of these three fields.

Part of the thesis is concerned with studies of low energy tests of quantum gravity, and beyond. One such test aims at detecting entanglement between masses in a superposition as a result of gravitational interaction. We detail a quantum optics simulation of this effect and report on the results. We then derive from first principles the quantum phases involved in this quantum gravity experiment. We obtain an exact formula that improves on the current literature, as it takes into account relativistic retardation and can be computed for general trajectories and arbitrary number of particles. We also propose an experiment to test the discreteness of time. This test involves a single mass in a superposition of paths in a weak gravitational field. We assess its experimental feasibility and find it in a similar range as the low energy quantum gravity experiments proposed in the literature.

The rest of the thesis focuses on more conceptual aspects of space and time. We study the origin of the time-orientation of the operational formulations of quantum mechanics, which is in tension with the time-reversal symmetry of the rest of fundamental physics. We argue that the formalism is time-oriented, not because the fundamental physics is time-oriented, but because it is designed to study situations created by time-oriented macroscopic systems. Finally, we explore the consequences of admitting relative facts in the ontology of quantum mechanics, as proposed by the relational interpretation of quantum mechanics, and as hinted by a recent no-go theorem. We argue in particular that this clarifies the measurement problem and Wigner's friend-type scenarios, and allows to explain the emergence of the shared macroscopic world in which humans operate.

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List of works

This thesis is based on the following articles developed with several collaborators.

- Andrea Di Biagio and Carlo Rovelli. "Stable Facts, Relative Facts". *Foundations of Physics* 51(1) (2021), page 30. DOI: `10/gm7w6w`. arXiv: `2006.15543`
- Andrea Di Biagio, Pietro Donà, and Carlo Rovelli. "The Arrow of Time in Operational Formulations of Quantum Theory". *Quantum* 5 (2021), page 520. DOI: `10/gm7w6n`. arXiv: `2010.05734`
- Marios Christodoulou, Andrea Di Biagio, and Pierre Martin-Dussaud. "An Experiment to Test the Discreteness of Time". 2020. arXiv: `2007.08431`
- Andrea Di Biagio and Carlo Rovelli. "Relational Quantum Mechanics Is about Facts, Not States: A Reply to Pienaar and Brukner". 2021. arXiv: `2110.03610` (to appear in Foundations of Physics)
- Marios Christodoulou, Andrea Di Biagio, Markus Aspelmeyer, Časlav Brukner, Carlo Rovelli, and Richard Howl. "Locally Mediated Entanglement through Gravity from First Principles". 2022. arXiv: `2202.03368`
- Emanuele Polino, Davide Poderini, Beatrice Polacchi, Iris Agresti, Gonzalo Carvacho, Fabio Sciarrino, Andrea Di Biagio, Marios Christodoulou, and Carlo Rovelli. "Photonic Implementation of Quantum Gravity Simulators" (in preparation)

Contents

Introduction

Current fundamental physics research concentrates on radically unfamiliar scales. Particle colliders explore regimes of the tiny and fast, while cosmology studies the unfathomably large and distant. When experiment and data collection are possible, the machinery involved is so sophisticated that it requires international collaborations of scientists and engineers numbering in the hundreds. Research in what is arguably the biggest open problem in fundamental physics—the search for a quantum theory of gravity—has been almost exclusively a theoretical effort for about 90 years. Doing experiments in quantum gravity is, to put it mildly, hard. The fundamental constants c, $\hbar$ and G offer us a dimensional estimate of the scales involved in a *bona fide* quantum gravitational phenomenon.

$$l_{\mathrm{P}} = \sqrt{\frac{G\hbar}{c^3}} \approx 10^{-35}\ \mathrm{m}, \qquad t_{\mathrm{P}} = \sqrt{\frac{G\hbar}{c^5}} \approx 10^{-44}\ \mathrm{s}, \qquad E_{\mathrm{P}} = \sqrt{\frac{\hbar c^5}{G}} \approx 10^{16}\ \mathrm{TeV}.$$

These quantities define the planck scale, and current technological capabilities are far from probing these scales directly. This can occasionally create a sense of disillusionment in the enterprise of quantum gravity. To this day, there is no genuine experimental evidence for quantum gravity, even though there are several candidate theories.

There is one field of fundamental physics that is an outlier in this respect, where theory and experiment go hand in hand and advances, both theoretical and experimental, are still being done by small groups. This is the field that studies the mathematical structure of quantum theory itself. This field consists of two intermingling and cross-pollinating communities: the quantum information and quantum foundations communities. One studies the information processing capabilities afforded by quantum physics and the other explores the mathematical and physical foundations of quantum mechanics. Historically, the rising interest in quantum foundations was heralded by the derivation of Bell's first theorem in the 1960s. Researchers started to ask in what ways the puzzling properties of quantum systems can be used illuminate reality, and to seek ways to employ these features for technology. In the last decades, the field of quantum information has been growing rapidly, with private companies and public institutions competing to build larger programmable quantum computers, with the promise of actual quantum advantage over classical computers. Experimentalists are constantly striving to achieve larger and longer lasting superpositions and developing new ways to spread entanglement to more systems and controlling them without losing coherence.

The increase of quantum control over larger systems has already benefited the other branches of physics. Today, it has enabled high precision tests of general relativity, such as the detection of gravitational waves, the measuring of gravitational time-dilation, and the quantification of gravitational acceleration due to minuscule masses. Soon, it might open a window into quantum gravity. Let us come back to the planck scale. One of the dimensions stands out: the planck mass

$$m_{\mathrm{P}} = \sqrt{\frac{\hbar c^2}{G}} \approx 20\ \mu\mathrm{g}$$

is just slightly lighter than a human hair. It is about the mass of 10^{17} silicon atoms. While we are still orders of magnitude away from achieving quantum superpositions of systems of this mass, the gap narrows. In the last few years, researchers have been proposing experimental setups featuring slow-moving quantum systems, designed to verify the non-classical nature of the gravitational field. Two masses, each in a superposition of position, become entangled by interacting with each other gravitationally. The effects are more pronounced the closer the mass of the involved quantum system is to the planck mass. Thus, the quantum gravity scale is being approached by the methods of fundamental research in quantum theory, not by smashing high-energy beams with each other, but by delicately isolating systems and allowing them to evolve, undisturbed, in superposition.

The work reported in this thesis concerns topics broadly in the intersection of the fields of quantum gravity, quantum information (QI) and quantum foundations (QF). Part I, introduces, in turn, each of these three fields. Chapter 1 discusses quantum gravity. In particular, we look at quantum gravity as a perturbative field theory, an effective theory which is predictive until very high energy scales. We also discuss the proposals to test quantum gravity in the low energy regime, focussing on the experiment aimed at detecting a specific prediction of quantum gravity, namely gravitationally mediated entanglement (GME). This has the potential of being the first experimental test of a genuine quantum gravity effect. Something else makes these tests interesting: there is a theorem from quantum information theory that a classical system cannot mediate the creation of entanglement. In chapter 2, we talk about quantum information theory. We introduce the operational formulation of quantum mechanics used in QI/QF research, and talk about some key concepts relevant to these experiments. After introducing the framework of generalised probabilistic theories, we reproduce a proof of the theorem mentioned above that makes GME so interesting. In chapter 3, we talk about quantum foundations. We see how the generalised probabilistic theories have been used to derive the Hilbert space structure of quantum mechanics starting from physical axioms about information processing. We then discuss the measurement problem and some of the interpretations of quantum mechanics that feature in the rest of the thesis. We also present results from the field of experimental metaphysics. These are no-go theorems that constraints our ultimate pictures of reality. Bell's theorems put strains on our notions of causality, while a recent theorem by Bong *et.al.* puts under stress our notion of objective reality. These three chapters do not attempt to cover exhaustively these fields, but are meant to provide motivation and context for the rest of the thesis, parts II to IV, which feature original research.

The start of the journey is the closest to experiment. In part II, we study the GME experiment in detail. In fact, in chapter 4, we propose a simulation of the experiment, which is being implemented on a photonic platform in Sapienza. The simulation embeds the evolution of the systems involved in the GME experiment in logical quantum circuits, and the circuits are then implemented using photonic degrees of freedom. Unfortunately, the experimental part of this project has been critically slowed down by the pandemic. nevertheless, we are able to report some amount of experimental data. Chapter 5 is more theoretical: we derive from first principles the exact formula for the quantum phases developed by the masses in the GME experiment. Extant derivations of the effect make use of a static approximation, which obscures the dynamical role played by the geometry in the experiment. We use linearised quantum gravity to derive the formulas to compute the phases for arbitrary trajectories. The phases show retardation effects due to the finite speed of propagation of perturbations in the geometry.

In part III, we talk about time. Chapter 6 proposes an experiment, while chapter 7 is a theoretical discussion. In chapter 6, inspired by the GME setup, we detail an experiment that could allow to detect a hypothetical discreteness of time of the order of a planck time. This is based on the realisation that quantum phases due to gravity can be understood in terms of differences in proper time. Weak gravitational field gradients correspond to small differences in proper time, which can reach planckian size. We derive experimental requirements to detect such a hypothetical discreteness of time and find, perhaps surprisingly, that these are not too far removed from current technological trends. While there is no unambiguous prediction of time-discreteness in the major approaches to quantum gravity, this is a previously untested regime of gravity and, as such, worth exploring. In chapter 7, we discuss a main point of tension between the gravity and the QI/QF communities: the status of time-reversal symmetry. Fundamental theories of mechanics are time-reversal symmetric, while the operational formulation of quantum theory is starkly time-oriented. This might be a problem when trying to extend this formulation to the search of a quantum theory of gravity. We argue that the origin of this time-asymmetry is to be found in the main domain of application of the operational formulation: laboratory physics, where time-oriented decision-making agents are manipulating the systems.

The end of the journey, part IV, is the most philosophical. Classical physics, is based on the existence of an objective reality, were facts happen and are absolute. Quantum mechanics complicates things, allowing objects to have indefinite properties. Nevertheless, most interpretations of quantum mechanics still assume the existence of a shared, objective reality. But the theorem by Bong *et.al.* mentioned above forces us to re-examine the notion of objective reality. We discuss aspects of the relational interpretation of quantum mechanics, which proposes to solve the measurement problem by introducing the notion or relative facts. According to the relational interpretation, facts happen at every interaction between any two systems, but that these facts are relative only to the systems involved in the interaction. In chapter 8, we discuss this idea in detail and see how to connect it with the observed world. In particular, we show how decoherence allows a large class of relative facts to become *stable*, meaning that their relative nature can be ignored. In chapter 9, we further explore the consequences of the relativity of facts, especially as it pertains to the analogy with special relativity and to the degree that objectivity can be achieved in the world of relative facts.

Part I

Some background

Chapter 1

Quantum Gravity

Why paradox? Because Einstein's equation says "this is the end" and physics says "there is no end."

Misner, Thorne and Wheeler [176]

1.1 Why we need it

General relativity (GR) unifies gravitational and inertial phenomena, explaining them as features of how matter interacts with the geometry of spacetime. Its dynamical content is expressed[1] by the Einstein Field Equations (EFE),

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G T_{\mu\nu}. \tag{1.1}$$

relating the *metric* field $g_{\mu\nu}$, which encodes the geometrical properties of spacetime, with the *energy-momentum* tensor field $T_{\mu\nu}$ of matter

The Einstein field equations not only managed to resolve a known tension between Newton's law of gravitation and the observation of the precession of Mercury's orbit, they have also predicted a number of surprising phenomena. A few examples are gravitational time-dilation, gravitational lensing of light, and the existence of gravitational waves. These phenomena have been experimentally detected, and the predictions of the theory are in exquisite quantitative agreement with the data. General relativity can be applied from the scale of a human cell [156] to the universe as a whole.

1.1.1 A collapse problem

Let us focus on the more troubling (and exciting) consequences of two more predictions of GR. The existence of black holes and the impossibility of a static universe were early predictions of the theory and, although unexpected, have been experimentally confirmed using astronomical data. They directly point to regimes where the theory fails.

[1] We take units in which $c = 1$ in this section.

The black hole is described by the first ever discovered exact solution of the EFE: the Schwartzchild metric [89, 237, 238]: a static, rotationally symmetric vacuum solution. A black hole is surrounded by an *event horizon*, a surface that cannot be crossed from the inside towards the outside. At the centre of the black hole lies a curvature singularity, a region of spacetime in which coordinate-independent quantities like the Kretschmann scalar $R^{\mu\nu\alpha\beta}R_{\mu\nu\alpha\beta}$ diverge. While this implies the theory cannot make sensible predictions there, the theory also says these places can be reached: while the event horizon shields the outside region from the singularity, a body falling freely into the black hole will reach the singularity in a finite proper time. For a while, black holes were thought of as the "unicorns" of theoretical physics, too strange to actually be realised in nature. It was hoped that they were a pathology of the high degree of symmetry of the ansatz and that they could not come about as a result of normal astrophysical evolution. However, a series of theoretical results, such as Oppenheimer's and Volkoff's [191], showed that bodies above a certain mass will eventually collapse inside their event horizon. Penrose famously showed [199] that the curvature singularity is not an artefact of the high symmetry of the Schwartzchild metric, but that it is inevitable given mild assumptions about the matter. Today, black holes are thought to be ubiquitous in the universe. Gravitational wave observatories have detected numerous events perfectly consistent with the merger of two black holes [160], or of a black hole and a neutron star [70]. Recent efforts lead to the reconstruction of an image of the extremely compact and massive object at the centre of the M87 galaxy that looks exactly as a black hole event horizon predicted by general relativity [72].

The other way that GR predicts its own demise is at cosmological scales. When considering the universe at its largest scales, one can approximate the density of matter and radiation as essentially constant at all places. One is led to consider a spacetime that is spatially homogenous and isotropic. The approximation reduces the EFE to a set of equations governing how the scale factor of the universe evolves in time. The scale factor determines how far two typical neighbouring galaxies are. The theory predicts that the scale factor cannot be constant: at most moments it is either increasing or decreasing, all galaxies are either getting closer or farther away from each other. Observational data show that galaxies are indeed receding, faster the farther away they are. Observations of the cosmic microwave background are consistent with the Einstein equations with a small positive cosmological constant Λ and cold dark matter [71]. Using this model to extrapolate further into the past, the EFE lead to a curvature singularity in the far past, another point where the theory cannot be trusted with its predictions. Hawking [127] showed that initial curvature singularities are quite general features of expanding universes.

These two facts about the universe indicate that GR cannot tell the full story for gravity. Something currently unknown must happen at the centre of black holes and at the beginning of time.

1.1.2 Hard to modify

One way to resolve these singularities is to modify the equations of GR. Perhaps one can find new equations that reproduce most predictions of GR in known regimes, but somehow avoid these singularities. This is the programme of *modified gravity*, an active field of research with several internal directions [66, 184]. Many of the modified gravity theories lead to falsifiable predictions, and indeed a number of modified gravity theories are falsified by current observational data. We will not attempt an overview of the subject, electing instead to offer one explanation as to why the research programme is hard.

The Einstein field equations are considerably constrained by the two principles that guided Einstein in the formulation of his theory [269]:

- **Principle of general covariance**: The equations of physics must take the same form in all coordinate systems.
- **Einstein equivalence principle**: Physics is arbitrarily close to being inertial in any sufficiently small spacetime region.

The first principle essentially implies that equations have to come in the form of *tensor field* equations, while the second implies that spacetime must have the geometry of a Lorentzian manifold. Following the analogy from Coulomb's law to Maxwell's equations, one is led to look for a relativistic generalisation of Poisson's equation for the gravitational potential

$$\nabla^2\Phi = 4\pi G\rho. \tag{1.2}$$

The mass density ρ is promoted to the energy-momentum tensor $T_{\mu\nu}$. The stress energy tensor is symmetric in its two indices. It should also be conserved by the covariant derivative, so to express the conservation of energy in small volumes according to the equivalence principle. Thus, the left hand side of the relativistic version of the equation above then should feature a conserved symmetric tensor, containing terms with, at most, second derivatives of the metric. The only such tensor is

$$\alpha(G_{\mu\nu} + \Lambda g_{\mu\nu}), \tag{1.3}$$

for some constants α and Λ, where

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R \tag{1.4}$$

is the Einstein curvature tensor, which contains terms with first and second derivatives of the metric. The constants of proportionality α and Λ are then fixed to match observational data, namely, Newtonian gravity and the observed expansion of the universe.

The action principle offers another *a posteriori* way to see how natural and constrained GR is. If we ask for the action to be computed via a Lagrangian density, we are looking to find an action of the form

$$\int \mathrm{d}^4x\ L(x,g), \tag{1.5}$$

where L is some function. Einstein's two guiding principles almost completely fix the form of the action. General covariance requires we integrate a *scalar density*, that is, that we write the action as

$$\int \mathrm{d}^4x\ \sqrt{-g}\mathcal{L}, \tag{1.6}$$

where $\sqrt{-g}\mathrm{d}^4x$ is the invariant volume element and $\mathcal{L}$ is some scalar. Einstein's equivalence principle then asks us to build this scalar from the metric only. In analogy with other field theories, we need kinetic terms with two derivatives of the metric to have any meaningful dynamics. There is essentially only one scalar formed with two derivatives of the metric: the Ricci scalar R. This argument leads us straight to the Einstein-Hilbert action

$$S_{\mathrm{EH}} \propto \int \mathrm{d}^4x \, \sqrt{-g}R. \tag{1.7}$$

Extremising this action leads to the vacuum EFE without cosmological constant. Dimensional analysis also tells us that the proportionality constant has to be proportional to $1/G$. One can fix the proportionality constant by coupling gravity with matter. The two principles also fix the shape of the matter action to

$$S_{\mathrm{M}} \propto \int \mathrm{d}^4x \, \sqrt{-g}\mathcal{L}_{\mathrm{M}}, \tag{1.8}$$

where $\mathcal{L}_M$ depends on the metric in such a way that it reduces to the flat spacetime Lagrangian when $g_{\mu\nu} \to \eta_{\mu\nu}$. This is can be done by the minimal coupling procedure, replacing partial derivatives by covariant derivatives. The action

$$S_{\mathrm{EH}} + S_{\mathrm{M}} = \int \mathrm{d}^4x \, \sqrt{-g} \left[\frac{1}{16\pi G} R + \mathcal{L}_{\mathrm{M}} \right] \tag{1.9}$$

leads to the EFE (1.1). Finally one might add the cosmological constant by adding a constant potential term to the action:

$$S_{\mathrm{cosmo}} = -\int \mathrm{d}^4x \, \sqrt{-g}\Lambda. \tag{1.10}$$

Seen as the two guiding principles of GR put a lot of constraints on the equations, modifying the EFE by adding terms on the left or right hand side of the EFE has the potential effect of spoiling the principles of general covariance and the equivalence principle. The action principle does suggest a straightforward modification of gravity, by replacing the Ricci scalar R as a lagrangian density by some function $f(R)$.

Of course the principles might not hold in all regimes, but since GR was so successful and offers a good explanation of physics, rather than just fitting data, it is arguably a good idea to take them seriously and use them as a guide to developing new theory.

1.1.3 Another collapse offers a hint

At the end of the XIXth century, another jewel of classical physics was confronted with a similar problem. Maxwell's field equations predict that two point particles of opposing charge cannot not form a stable configuration. The two charges, accelerated by their mutual attraction, would emit radiation, loose energy, and thus fall closer towards each other. This process leads them to emit an infinite amount of radiation in a finite time. Experiments then showed that atoms are composed of negatively charged particles surrounding a positively charged nucleus, many times smaller than the radius of the atom itself [31, 231]. Exactly the kind system that Maxwell's equations predicted to be unstable! Decades of theoretical effort to explain this paradoxical stability of matter, together with many more puzzling phenomena such as the spectral lines in emission and absorption spectra, the photoelectric effect, and the observation of discrete values of angular momentum, eventually lead to the formulation of quantum mechanics, quantum field theory, and quantum electrodynamics [195].

This historical sketch suggests a hint for the resolution of GR's singularities: perhaps quantum theory can fix them. The quantum theory of matter, on its own is not enough to prevent the singularities, as the gravitational pressure can beat any pressure caused by known quantum effects in condensed matter. But a full quantum theory of gravity could provide a solution to the gravitational collapse and furnish us with predictions about what happens at these most extreme events in our universe.

There is another reason that the community wishes for a quantum theory of gravity. While the EFE (1.1) couple the metric field with the energy-momentum tensor field of matter, matter is described by quantum mechanics and its energy-momentum density is not a real-valued tensor field $T_{\mu\nu}$ with a single definite value at any location, but a linear-operator-valued tensor field $\hat{T}_{\mu\nu}$ acting on the Hilbert space associated with matter. Thus in light of quantum mechanics, the EFE can only make sense if either

- the right hand side is a real-valued tensor field, for example $\langle\hat{T}_{\mu\nu}\rangle$, or
- the left hand side is also some operator $\hat{G}_{\mu\nu}$ acting on an appropriate Hilbert space.

The first option, where the angle brackets represent the expectation value of $\hat{T}_{\mu\nu}$, is often called *semi-classical* gravity. It offers correct predictions when the size of the quantum fluctuations are small, *i.e.* for most astrophysical applications. But it has otherwise maladapted features.[2] For example, nothing forces $\partial_\mu\langle\hat{T}_{\mu\nu}\rangle$ to vanish, while $\partial_\mu G^{\mu\nu}$ vanishes identically. So it can't work as a fundamental theory.

The other option would be the development of a quantum theory of gravity. Contrary of what is generally said, such a theory exists. Unfortunately, it does not tell us much about the singularities.

1.2 General relativity as a quantum field theory

When first encountering the subject of quantum gravity, one is often offered a statement to the effect that general relativity and quantum field theory are irreconcilably different, and that it is impossible to "quantise" general relativity. This is in fact a false statement. Quantum field theory and general relativity are not in contradiction with each other. The opposite is true: there is a well-defined—and predictive—quantum field theory of general relativity. Actually, there is a sense in which quantum field theory *predicts* general relativity. The structure of relativistic quantum field theory is rigid enough that it is possible to show that the only theory that couples relativistically to energy density is, in fact, general relativity. Let's see why.

[2]Depending on one's preferred solution to the measurement problem, semiclassical gravity has been empirically disproven [194].

1.2.1 Quantum field theorists could have discovered GR

Let us imagine a world where physicists have developed special relativity and quantum field theory, but that nobody has succeeded in obtaining a relativistic classical theory of gravity and instead physicists still relied on Newtonian gravity. An argument has been made, notably by Feynman [100] and more recently by Schwartz [236], that quantum field theorists in this imagined world could come up with general relativity, and they could do so while bypassing all arguments about geometry. Here, we sketch the main points of such an argument, following first [236] then [100].

The formulation of a quantum field theory is generally centred around an action S, computed as the integral of a lagrangian density over a region of spacetime. The lagrangian density at a point is generally a function of the fields and their derivatives at that point. The fields might carry representations of various symmetry groups, and the theory is said to have a given symmetry if the action is left invariant under those transformations. In particular, compatibility with special relativity requires the action to be invariant under the group of Poincaré transformations (rotations, boosts, and translations). It is then informative to classify the various fields according to the Poincaré group representation they carry. Fundamental fields carry *irreducible representations*, and fundamental particles are identified with the quantised excitations of modes of the corresponding field. Irreducible representations are labelled by two Casimir operators, the mass m and angular momentum j.

Relativistic quantum field theory makes quite general predictions. For example, the energy between two sources resulting from their interaction with a field of mass m and angular momentum j depends on the distance r between the sources as

$$U \propto \frac{e^{-\alpha m r}}{r}, \tag{1.11}$$

where α is some dimensionful constant. Thus, the *range* of an interaction depends on the carrier's mass m. The *sign* of the energy U depends instead on the angular momentum j of the carrier. The energy between two positive charges is positive for odd integer values of j, while it is negative for even integer values.

We summarise the known properties of gravity: it is a long range, attractive force that couples to mass, or, more properly, mass density ρ. From relativity, we know about the mass-energy relation, and that the relativistic quantity corresponding to ρ is $T^{\mu\nu}$, which in the Newtonian regime reduces to ρ. According to quantum field theory, forces are mediated by fields that carry representations of the Poincaré group with integer angular momentum j and a certain mass m. The long range of gravity implies the mass is close to 0. The fact that the force is attractive rules out odd integer spins. So one is lead to study interactive field theories with $j = 0, 2, \dots$ fields. Theories with higher spin are progressively harder, so one starts with the lowest orders first.

Successful quantisation depends on having a classical lagrangian formulation of the field theory. Terms in the lagrangian have to be Lorentz scalars, the easiest way to do so is to build them by contracting tensors. Thus a Lorentz, invariant minimal coupling term for a $j = 0$ field ϕ and mass-energy is ϕT, where $T = \eta_{\mu\nu} T^{\mu\nu}$. This does not lead to a theory that agrees with experiment [269], so one moves on to spin-2.

Fierz and Pauli have derived the unique lagrangian for a massive spin-2 field $h_{\mu\nu}$ [102]. In the massless limit, the lagrangian is:

$$\mathcal{S}_{\mathrm{FP}} = \frac{1}{4}h_{\mu\nu}\Box h^{\mu\nu} - \frac{1}{2}h_{\mu\nu}\partial^{\mu}\partial_{\alpha}h^{\nu\alpha} + \frac{1}{2}h\partial_{\mu}\partial_{\nu}h^{\mu\nu} - \frac{1}{4}h\Box h. \tag{1.12}$$

A generic generic way to couple to this field would be a term such as $h_{\mu\nu}S^{\mu\nu}$, for some symmetric tensor $S^{\mu\nu}$ composed from the rest of the fields. For $h_{\mu\nu}$ to be a massless spin-2 field, the only physical components should be the transverse ones, in other words, the action should not care about the longitudinal components of $h_{\mu\nu}$ and should be invariant under a change:

$$h_{\mu\nu} \longmapsto h_{\mu\nu} - \partial_{\mu}\alpha_{\nu} - \partial_{\nu}\alpha_{\mu}. \tag{1.13}$$

For the coupling term $h_{\mu\nu}S^{\mu\nu}$, this requires that the tensor $S^{\mu\nu}$ be conserved by the equations of motion, *i.e.* that $\partial_{\mu}S^{\mu\nu} = 0$. So it happens that a massless spin-2 field $h_{\mu\nu}$ can couple to a conserved energy-momentum tensor $T^{\mu\nu}$. In fact, it seems that it is *made* to couple to energy-momentum.

The resulting action

$$S^1 = \int \mathrm{d}^4x \left[\mathcal{L}_{\mathrm{FP}} + \mathcal{L}_{\mathrm{M}} - \lambda h_{\mu\nu}T^{\mu\nu}\right], \tag{1.14}$$

when used to first order in $h_{\mu\nu}$, already quantitatively describes a number of gravitational phenomena. For example, it yields linear equations for $h_{\mu\nu}$ and can be used to compute the motion of test particles given a fixed $h_{\mu\nu}$. This allows to fix the free parameter λ. This theory reproduces Newtonian gravity and makes quantitatively correct predictions of gravitational time dilation and deflection of relativistic particles by weak gravitational fields. This theory also predicts the existence of gravitational waves, and allows to figure out the design of instruments to detect them.

However, the theory does not account for the precession of the perihelion of Mercury. Also, it cannot be used consistently to second order, as the equations of motion for the matter do not conserve $T^{\mu\nu}$ to second order. One way to move forward would the following. Notice that gravitational waves carry energy and momentum, so that the gravitational field should also be a source of the gravitational field. But notice also that the equations of motion obtained by varying (1.15) with respect to $h_{\mu\nu}$ only have $T^{\mu\nu}$ as a source. Thus, one might try to add some term F to $\mathcal{L}_{\mathrm{FP}}$ so that variations with respect to $h_{\mu\nu}$ yield an equation of motion that has as a source a tensor $T^{\mu\nu} + \chi^{\mu\nu}$ where $\chi^{\mu\nu}$ contains terms quadratic in $h_{\mu\nu}$. Asking that the new source be conserved ($\partial_{\nu}(T^{\mu\nu} + \chi^{\mu\nu}) = 0$) to second order fixes F uniquely. The resulting theory,

$$S^2 = \int d^4x \left[\mathcal{L}_{\mathrm{FP}} + F + \mathcal{L}_{\mathrm{M}} - \lambda h_{\mu\nu}T^{\mu\nu}\right], \tag{1.15}$$

can be consistently used to second order. It is in quantitative agreement with the observed the perihelion shift of Mercury, so this theory is in agreement with all the first famous tests of general relativity.

However, the theory is not consistent to *third* order in $h_{\mu\nu}$ and above. One can proceed the same way as above, adding term after term to the action and requiring conservation of the resulting source. This becomes harder and harder, but, surprisingly there is a way to find a term that is analytical in $h_{\mu\nu}$ (so that it contains arbitrary order terms) and that leads to a self-consistent theory. The resulting action for the kinetic terms of $h_{\mu\nu}$ is then

$$S^\circ \propto \int d^4x \sqrt{-\det(\eta_{\mu\nu} + h_{\mu\nu})} R\left[\eta_{\mu\nu} + h_{\mu\nu}\right], \tag{1.16}$$

which is precisely the Einstein-Hilbert action for $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$.

Thus looking for a self-consistent theory of spin-2 field interacting with matter leads to the Einstein-Hilbert action. Deser [78, 79] in fact proved that this is the *only* such self-consistent theory.

1.2.2 The actual problem of quantum gravity

Quantum field theorists in our imagined world, having thus derived a classical action for a relativistic theory of gravity could go on and compute its consequences, hopefully stumbling upon the interpretation in terms of Lorentzian geometry. They would realise that the gauge invariance under transformations (1.13) is the linear version of general covariance. They would also discover the singularities predicted by the theory at the end of stellar evolution and at the beginning of time. So the next question they might ask is exactly the question we are asking: do quantum effects fix the collapses, like they fixed those of the atom?

Unfortunately, the answer is no. But the problem is not that the action (1.16) cannot be quantised. The theory obtained from this action is non-renormalisable, but this does not mean the theory is useless. Non-renormalisable theories are predictive because, up to a given energy cut-off, only a finite number of measurements have to be made to fix the free parameters, and then one can compute predictions for arbitrary experiments in that energy range. Indeed, because the planck mass is so large compared to other high energy physics scales, only the first few terms have played a role in experiments. In fact, it is possible to compute genuine quantum corrections to the orbit of mercury by considering the renormalised one loop correction to the graviton propagator, and this leads to a genuine quantum modification to the perihelion shift of mercury of one part to 10^{90} [236].

The actual problem is that perturbation theory breaks down at energy scales larger than planck mass. One needs to find a *different* theory that works at higher energy scales and that reduces to general relativity at lower energy scales. This sort of theory is called a *UV completion*, and the non-renormalisable theory is then the low energy effective field theory of its UV completion. History teaches that the UV completion of a theory can be quite different from the original effective theory. It can also be the case that a theory can have *multiple* UV completions. For example, the Schrödinger equation is the low energy limit of both the Klein-Gordon equation and the Dirac equation. This is also the case with quantum gravity. At the moment we have several candidate UV completions, the most famous ones being string theory [277] and loop quantum gravity [223, 230]. String theory continues with the perturbative approach, looking for a UV finite perturbative theory, while loop quantum gravity is based on a non-perturbative approach, quantising the whole geometry $g_{\mu\nu}$ rather than the perturbation $h_{\mu\nu}$. For an overview of the approaches, their conceptual differences and difficulties, see [42].

1.3 Why do we need it, again?

Since the low energy theory is predictive, the differences between the UV completions only appear at the scales where the low energy theory breaks down. As we have seen, the perturbative quantisation of general relativity yields predictions that are exceedingly similar to general relativity itself. The most straightforward ideas to test the quantum nature of gravity, such as reaching the non-perturbative regime of particle physics or detecting quantised gravitational radiation seem completely infeasible [92, 220]. Cosmological evidence from the CMB or the distribution of primordial black holes has been inconclusive thus far. This absence of any positive evidence for a quantum theory of gravity is part of the difficulty of settling on a specific UV completion.

Additionally, the absence of any quantum gravitational effect has lead some to question the very idea that gravity is mediated by a quantum field [43, 190, 200]. While there are *theoretical* arguments for the need of a quantum theory of gravity, there is no direct *empirical* reason for it. This situation might change in the near future.

Over the recent years, a number of experimental proposals aim to detect an effect that can only be explained by a genuinely non-classical gravitational field. The proposals include the generation of entanglement between two systems due to gravitational interaction alone. Proposals include two particles in adjacent matter-wave interferometers [34, 164], two particles cooled down to the ground state of a harmonic trap and then dropped side-by-side[149], and entanglement between a resonator and a particle in a trap [44]. Another proposal aims at detecting a different effect, the development of non-gaussianity in a Bose-Einstein condensate [140].

Two things are worth pointing out now. First, these are not simply *gedanken* experiments: even though their feasibility is still being assessed, these are actual experimental proposals. The authors of [34] have been publishing several studies regarding the experimental practicalities of their proposal [167, 253, 256], while [149] went through a considerable revision during peer-review. And while the "table-top" label originally attached to these proposals was most likely too optimistic, they seem within reach of feasible technology [8]. Second, the interest around these experiments is partially justified by results from quantum information theory. The experiments seek to detect an effect that is predicted by linearised quantum gravity. A positive result, would thus corroborate this theory, and falsify theories that do not predict this result. However, arguments from quantum information theory [112, 165] imply that a *whole class* of possible theories is ruled out by these experiments, namely, any theory in which the gravitational field does not have non-commuting variables. We will overview one of these results in detail in sections 2.4 and 2.5.

These experiments will not be able to distinguish between different UV completions of linearised quantum gravity. They will, however, definitely rule out either linearised quantum gravity, or the hypothesis that gravity is mediated by a classical field.

1.4 A low energy test of quantum gravity

Now, let us describe an idealised version of the matter-wave experiment proposed in [34]. This experiment aims at detecting gravity mediated entanglement (GME).

1.4.1 GME: experimental setup

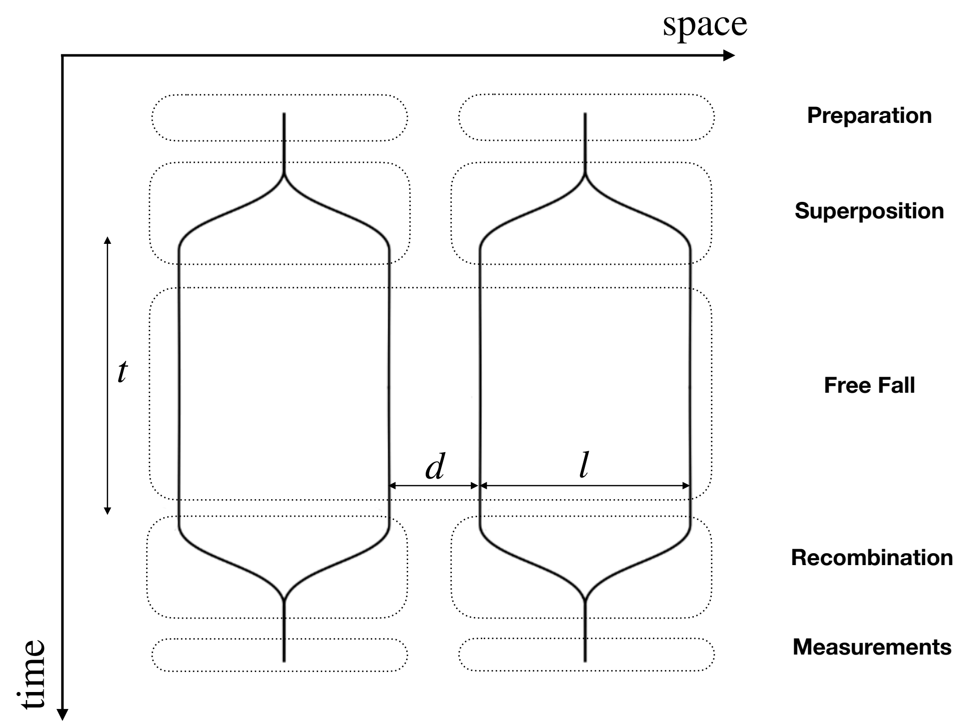


Figure 1.1. Sketch of the simulated experimental setup as proposed in [34]. Two masses are set in a spin-dependent superposition next to each other and allowed to interact only via gravity.

The experiment features two adjacent interferometers, each traversed by a mass with an embedded spin. The interferometers put each particle in a spin-dependent superposition of positions, then the particles are allowed to interact only via gravity for a time t before the interferometers undo the superposition. Finally, the spin on the particles is measured.

Let us go over the experiment, breaking it down in five stages, see figure 4.1. Let $|\uparrow\rangle, |\downarrow\rangle$ denote normalised states of the spins aligned and anti-aligned to the z axis. Let $|C\rangle, |L\rangle$, and $|R\rangle$ denote normalised states in the centre-of-mass Hilbert space of each particle, peaked at the centre, left, and right branches of their interferometer. We will write down the total state up to an overall normalisation factor.

At the Preparation stage, the particles of mass m are set at the centre of their interferometers, with the spin aligned to the x axis. The state of the system is:

$$\begin{aligned} |\psi\rangle_{\mathrm{P}} &= (\,|\uparrow\rangle + |\downarrow\rangle\,) \otimes (\,|\uparrow\rangle + |\downarrow\rangle\,) \otimes |CC\rangle \\ &= (\,|\uparrow\uparrow\rangle + |\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle + |\downarrow\downarrow\rangle\,)\,|CC\rangle \end{aligned} \tag{1.17}$$

During the Superposition, stage a series of EM pulses modifies the position of the masses depending on z-component of the spin state resulting in each mass being in a spin-dependent path superposition, at rest at a distance $l/2$ from their initial position. Assuming that the two interferometers are identical, and exploiting the phase ambiguity in defining $|L\rangle$, the state after superposition is:

$$|\psi\rangle_{\mathrm{S}} = |\uparrow\uparrow\rangle\,|LR\rangle + |\uparrow\downarrow\rangle\,|LL\rangle + |\downarrow\uparrow\rangle\,|RR\rangle + |\downarrow\downarrow\rangle\,|RL\rangle\,. \tag{1.18}$$

At this point, the centre-of-mass of each particle is entangled with its spin, but there is no entanglement between the two particles.

Now, the particles are in free fall for a time t. The experiment is designed in such a way that the only relevant interaction between the two particles is the gravitational force. Notably, keeping the Casimir-Polder interaction at bay sets a lower bound for the distance of closest approach d between the masses. The EM pulses of the Superposition stage also had to take care to "hide" the spin deep into the particle [34]. These experimental considerations aside, the states $|R\rangle$ and $|L\rangle$ act to a very good approximation as energy eigenstates during the relatively short time t of the experiment [54], and they will thus simply each accumulate a phase. At the end of the Free Fall stage, the state is

$$|\psi\rangle_{\mathrm{FF}} = e^{i\phi_{LR}}\,|\uparrow\uparrow\rangle\,|LR\rangle + e^{i\phi_{LL}}\,|\uparrow\downarrow\rangle\,|LL\rangle + e^{i\phi_{RR}}\,|\downarrow\uparrow\rangle\,|RR\rangle + e^{i\phi_{RL}}\,|\downarrow\downarrow\rangle\,|RL\rangle \tag{1.19}$$

For generic values of the phases $\phi_{LL}, \phi_{LR}, \phi_{RL}$, and ϕ_{RR}, the two particles are now entangled.

The Recombination stage now undoes what was done by the Superposition stage. Again ignoring a possible global phase, the state of the system is now:

$$|\psi\rangle_{\mathrm{R}} = \left(e^{i\phi_{LR}}\,|\uparrow\uparrow\rangle + e^{i\phi_{LL}}\,|\uparrow\downarrow\rangle + e^{i\phi_{RR}}\,|\downarrow\uparrow\rangle + e^{i\phi_{RL}}\,|\downarrow\downarrow\rangle\right) \otimes |CC\rangle\,. \tag{1.20}$$

The net effect is that the centre-of-mass degree of freedom is not correlated with the spins anymore. Whatever entanglement was developed between the masses is now present in the spin degrees of freedom only.

Tracing away the positions of the particles, the state of the spins at the moment of measurement is then:

$$|\psi\rangle_{\mathrm{M}} = e^{i\phi_{LR}}\,|\uparrow\uparrow\rangle + e^{i\phi_{LL}}\,|\uparrow\downarrow\rangle + e^{i\phi_{RR}}\,|\downarrow\uparrow\rangle + e^{i\phi_{RL}}\,|\downarrow\downarrow\rangle\,. \tag{1.21}$$

Repeated runs of the experiment allow to study the final state of the spins, and verify the presence of entanglement.

1.4.2 Quantum gravitational phases

Let us now compute the phases that the particles develop during the Free-Fall stage.

The particles are light, and the perturbation of the metric due to their presence is weak enough so that we may indeed apply linearised quantum gravity. We assume that the particles are moving slowly at all times, and that the timescales of relevance are much larger than the distances between the particles in each branch, so that we may ignore transient effects due to the movement of the particles. In this way, in each branch, the metric perturbation is in a coherent state peaked around the classical static solution sourced by the masses.

Write as $|h_{XY}\rangle$ the state of the metric perturbation corresponding to positions $|XY\rangle$ of the particles. Then we can track the evolution of the states of the particles and metric by trivially replacing $|XY\rangle$ with $|XY\rangle\,|h_{XY}\rangle$ everywhere in the states (1.17) to (1.20) above. The phases are then due to the energy of such a configuration, which can be approximated to a good accuracy by the Newtonian potential energy of the configuration. Namely,

$$\phi_{XY} = -E_{XY}\frac{t}{\hbar}, \tag{1.22}$$

and thus

$$\phi_{LR} = \frac{Gm^2 t}{(d+2l)\hbar}, \qquad \phi_{LL} = \frac{Gm^2 t}{(d+l)\hbar} = \phi_{RR}, \qquad \phi_{RL} = \frac{Gm^2 t}{d\hbar}. \tag{1.23}$$

Then, up to a global phase and normalisation, the state for the spins at the moment of measurement is

$$|\psi\rangle_{\mathrm{M}} = |\uparrow\rangle\left(e^{i\tilde{\phi}_{LR}}\,|\uparrow\rangle + |\downarrow\rangle\right) + |\downarrow\rangle\left(|\uparrow\rangle + e^{i\tilde{\phi}_{RL}}\,|\downarrow\rangle\right), \tag{1.24}$$

where we have set $\tilde{\phi}_{LR} = \phi_{LR} - \phi_{RR}$ and $\tilde{\phi}_{RL} = \phi_{RL} - \phi_{RR}$. We can quantify the amount of entanglement between the spins by taking the square modulus of the overlap of the state in the parenthesis, which is

$$O = \frac{1}{2} + \frac{1}{2}\cos(\delta\phi), \tag{1.25}$$

with

$$\delta\phi = \tilde{\phi}_{LR} + \tilde{\phi}_{RL} = \frac{Gm^2 t}{\hbar}\left(\frac{1}{d} + \frac{1}{d+2l} - \frac{2}{d+l}\right). \tag{1.26}$$

Thus, for generic parameter values, quantum field theory of gravity predicts that the masses will get entangled.

Note that these phases were computed in the static approximation. These are the same phases one would get from an instantaneous newtonian interaction term

$$H_I = -\frac{Gm^2}{r} \tag{1.27}$$

in the total hamiltonian of the system. Indeed, this is how the computations in the original papers [34, 149, 164] were done. Some, most notably [6], have objected that this spoils the argument, as it is a direct interparticle interaction rather than a mediated one. In chapter 5, we will see how to derive these phases from first principles, using the perturbative quantum theory of gravity.

1.4.3 Experimental considerations

There are four main experimental parameters subject to various constraints: the mass of the nanoparticles m, the distance of closest approach d, the size of the superposition l, and the duration of the superposition t. One of the main constraints comes from decoherence: the masses have to be protected from interacting with the environment otherwise the entanglement between them will be hidden. Larger l and larger m will lead to faster decoherence, and t must be much smaller than the typical decoherence timescales. Another constraint comes from preventing interactions between the masses other than gravity: if the masses come too close to each other other, they could start interacting via Casimir-Polder type electrostatic effects. This will force an upper bound on d as a function of m. Other constraints will come from the specifics of the experimental setup, such as the strength of the fields required to create the superpositions and the size and control of the various moving parts of the apparatus.

Future experimenters will have to take all such constraints in consideration. Here, we will limit ourselves to some basic observations. We will see in the next section how to certify the presence of entanglement using an entanglement witness. For now, it suffices to know that entanglement is easier to detect for smaller overlap O; easiest for $O \approx 0$, so for $\delta\phi \approx \pi$. Thus, we can see (1.26) as defining an entanglement rate r as a function of the parameters l, d and m. We can take the approximation $l \ll d$, reflecting the realistic assumption that the delocalisation of a massive nanoparticles is going to be small. To leading order in l/d, this yields:

$$r = 2\frac{G}{\hbar}\frac{m^2}{d}\left(\frac{l}{d}\right)^2 = 2\left(\frac{m}{m_{\mathrm{P}}}\right)^2\left(\frac{l}{d}\right)^2\frac{c}{d} \tag{1.28}$$

The entanglement rate r is inversely proportional to d^3, so one should pick the smallest possible value of d. To avoid interactions other than gravity, this is $d \approx 200\ \mu$m. Thus, $c/d \approx 10^{12}\ \mathrm{s}^{-1}$, is an encouraging baseline. However, the other two factors force $r < c/d$. Decoherence makes it difficult to have larger values of l, assuming optimistically that $l/d \approx 1/100$ then yields

$$r \approx \left(\frac{m}{m_{\mathrm{P}}}\right)^2 10^8\ \mathrm{s}^{-1}. \tag{1.29}$$

Thus, we see that pushing full quantum control of masses approaching the planck mass for the particles in superposition holds the key to witnessing this quantum gravitational effects. In particular, to have for $r \approx 1\ \mathrm{s}^{-1}$, which requires the coherence to last at least as long, means $m \approx 10^{-4}\, m_{\mathrm{P}} \approx 10^{-12}$ kg.

These are rough[3] figurings, but they give an idea of the requirements on the experiments. The details of course will depend on the specifics of the setup.

As mentioned above, while a precise measurement of the entanglement rate can serve as a *quantitative* test for of quantum gravity, the simple detection of the presence of entanglement at the moment of measurement is an interesting *qualitative* test. Indeed, as we will see in the next chapter, detecting the presence of *some* entanglement is sufficient to conclude that the field mediating the interaction between the two masses is not a classical field. Thus one is interested in experimental protocols capable of verifying the presence of entanglement, which we will also cover in the next chapter.

[3]For reference, the parameters in the original proposal are $m \approx 10^{-14}$ kg, $l \approx 250\ \mu$m, $d \approx 200\ \mu$m, which yield $\delta\phi \approx 0.5$.

Chapter 2

Quantum Information Theory

Quantum mechanics is astonishingly simple—once you take the physics out of it!

Scott Aaronson [135]

Claude Shannon formalised the concept of *information* carried by a system as the number of perfectly distinguishable states it can be in and introduced the concept of a *bit* a system that can be in either of two perfectly distinguishable states [240, 241]. These states are generally denoted 0 and 1. The bit is the fundamental unit of information, as it can be used to represent the truth value of a given proposition. In other words, it can be the answer to any “yes or no” question. If the state of the bit is not known with certainty, one can assign a probability distribution $\{p_0, p_1\}$ over its two possible values, where $p_0 + p_1 = 1$. Note that a classical bit can be embodied by any number of systems, from coins to switches. This is an advantage, as it makes information an abstraction similar to the notion of numbers. Indeed, classical information theory is the study of how information can be manipulated, processed and transmitted, by considering *abstract* manipulations of strings of 0s and 1s.

Quantum information theory is similarly centred around the concept of a *qubit*: a quantum system that can be in two perfectly distinguishable states. A bit can be encoded in the value of the z-component of the spin of an electron, or its x-component, or the component along any axis in fact. Any two-valued observable will do. Once chosen, it will be called the *computational basis* and its two eigenstates will be called $|0\rangle$ and $|1\rangle$. A generic *pure state* will be given by a Hilbert space vector

$$|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle\,, \tag{2.1}$$

for two complex numbers α and β such that $|\alpha|^2 + |\beta|^2 = 1$. The two states

$$|\pm\rangle = \frac{1}{\sqrt{2}}\,|0\rangle \pm \frac{1}{\sqrt{2}}\,|1\rangle \tag{2.2}$$

are often used. The overall phase is irrelevant, and it follows that any qubit pure state can be labelled by two real numbers $(\theta, \phi) \in [0, \pi] \times [0, 2\pi]$ and ϕ as

$$|\psi\rangle = \cos\frac{\theta}{2}\ |0\rangle + e^{i\phi}\sin\frac{\theta}{2}\ |1\rangle\,. \tag{2.3}$$

Thus, the pure states of a qubit correspond to a two-dimensional sphere, known as the Riemann, or Bloch, sphere. Antipodes of the sphere correspond to perfectly distinguishable states.

Classic examples of qubits include the spin of an electron and the polarisation of a laser, but *any* two-dimensional subspace of a quantum system will do the job. Then quantum information theory studies the information processing that collections of such finite-dimensional quantum systems afford, assuming (perfect or limited) control of these systems. From this point of view, a system associated with a d-dimensional Hilbert space is equivalent to any other system associated with a d-dimensional Hilbert space. Quantum information theory is the theory of the general evolution of finite-dimensional systems given arbitrary manipulation powers. The protocol that are thus devised can then be implemented on a number of different substrates.

2.1 Operational formalism

The state vector representation is somewhat limited for the needs of quantum information theory. Dealing with multiple subsystems, classical information, and classical uncertainty naturally leads to consider *density operators* as representing the state of the system, and study the evolution of these operators. We now provide a brief review.

2.1.1 Density operators

A *density operator*, or *density matrix*, ρ is a trace 1, positive operator. The density operator is used to compute expectation values as follows. If the state of a system is ρ, then the expectation value of an observable A is

$$\langle A\rangle = \operatorname{tr} A\rho. \tag{2.4}$$

After finding the value a of the observable, one updates the state to

$$\rho \longmapsto \frac{|a\rangle\!\langle a|\,\rho}{\operatorname{tr}|a\rangle\!\langle a|\,\rho}. \tag{2.5}$$

During unitary evolution U of the system, the density operator evolves as

$$\rho \longmapsto U\rho U^\dagger. \tag{2.6}$$

One reason to use the density matrix is that it allows to encode classical uncertainty about the quantum state. If you believe that a system is in state $|\psi\rangle$ with probability p and in state $|\phi\rangle$ with probability $1-p$, then the expectation value for any observable A is given by

$$\langle A\rangle = p\,\langle\psi|A|\psi\rangle + (1-p)\,\langle\phi|A|\phi\rangle\,. \tag{2.7}$$

This can easily be cast in the form (2.4) by using a resolution of the identity:

$$\begin{aligned}\langle A\rangle &= \sum_i \left(p\,\langle\psi|i\rangle\langle i|A|\psi\rangle + (1-p)\,\langle\phi|i\rangle\langle i|A|\phi\rangle\,\right)\\ &= \sum_i \langle i|\,A(p\,|\psi\rangle\langle\psi| + (1-p)\,|\phi\rangle\langle\phi|\,)\,|i\rangle \\ \langle A\rangle &= \operatorname{tr}\rho A,\end{aligned} \tag{2.8}$$

where we have defined

$$\rho = p\,|\psi\rangle\langle\psi| + (1-p)\,|\phi\rangle\langle\phi|\,. \tag{2.9}$$

In general, a density operator can be used to compute the probabilities of measurements performed on statistical ensembles of states. Given a set of states $\{|\psi_n\rangle\}$, if a quantum system is with probability p_n in the state $|\psi_n\rangle$, then the statistics of measurements will be reproduced by

$$\rho = \sum_n p_n\,|\psi_n\rangle\langle\psi_n| \tag{2.10}$$

and the rules (2.4) and (2.5). Note that a given mixed ρ will correspond to multiple ensembles. Indeed, the computation above shows us that *different* ensembles represented by *same* density operator cannot be distinguished by any kind of measurement.

It is useful to draw a distinction between *pure* and *mixed* states. A pure state satisfies

$$\operatorname{tr}\rho^2 = 1, \tag{2.11}$$

while for a mixed state

$$\operatorname{tr}\rho^2 < 1. \tag{2.12}$$

By diagonalising the density operator, and using the property $\operatorname{tr}\rho = 1$, one sees that a pure state is always of the form $\rho = |\psi\rangle\langle\psi|$, while a mixed state will always reproduce the statistics of a statistical ensemble of states. Since there is a bijection between the Hilbert space rays and the pure density operators, the density operator formalism is a proper generalisation of the Hilbert space vector. Finally, let us mention the existence of a special density operator, the *mixed state*

$$\frac{1}{d}\sum_{i=1}^{d} |i\rangle\langle i|\,, \tag{2.13}$$

which represents the state of minimal information about a system.

2.1.2 Bloch ball

Let us consider as an example the state space for qubits. The space of density operators for a qubit is the set of 2×2 positive matrices. There is a neat way to visualise it. The 2×2 identity matrix

$$\mathbb{I} = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} \tag{2.14}$$

and the Pauli matrices

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \tag{2.15}$$

form a basis for self-adjoint 2×2 matrices. If $\vec{r}$ is a 3-component vector of unit norm, then the matrix

$$\frac{1}{2}\mathbb{I} + \frac{1}{2}\vec{r} \cdot \vec{\sigma}, \tag{2.16}$$

is a unit trace, positive operator. Thus, the space of density operators of a qubit is isomorphic to a 2 dimensional ball, the *Bloch ball*. The boundary consists of the pure states, and the inside the mixed states.

In the QI literature, the Pauli matrices are often represented as X, Y, and Z.

2.1.3 Partial trace

Density matrices are useful in quantum information theory also because they naturally come up in the study of multiple systems. Given two systems associated with Hilbert spaces $\mathcal{H}_1$ and $\mathcal{H}_2$, respectively, the Hilbert space of the combined system is given by the tensor product $\mathcal{H}_1 \otimes \mathcal{H}_2$ of the two Hilbert spaces. Given bases $\{|a\rangle\}$ and $\{|m\rangle\}$ for $\mathcal{H}_1$ and $\mathcal{H}_2$, the set of states $\{|am\rangle \equiv |a\rangle \otimes |m\rangle\}$ forms a basis for the Hilbert space $\mathcal{H}_1 \otimes \mathcal{H}_2$, and a generic pure state will be written as

$$\sum_{am} \alpha_{am} |am\rangle \,. \tag{2.17}$$

One might be interested in the statistics of measurements on $\mathcal{H}_1$ only. Say for example one is doing a projective measurement $\{P_k\}$ on the first subsystem, then the probability of obtaining the result k, given the state above is

$$\langle P_k \otimes \mathbb{I} \rangle = \sum_{mnab} \alpha^*_{am} \langle am| (P_k \otimes \mathbb{I}) \alpha_{am} |bn\rangle \,, \tag{2.18}$$

where $\mathbb{I}$ represents the identity on $\mathcal{H}_2$. One can rearrange the expression as

$$\begin{aligned} \langle P_k \otimes \mathbb{I} \rangle &= \sum_{mnab} \alpha^*_{am} \alpha_{bn} \langle a|P_k|b\rangle \langle m|n\rangle \\ &= \sum_{mab} \alpha^*_{am} \alpha_{bm} \langle a|P_k|b\rangle \\ \langle P_k \otimes \mathbb{I} \rangle &= \operatorname{tr} \sum_{mab} \alpha^*_{am} \alpha_{bm} P_k |b\rangle\langle a| \,. \end{aligned} \tag{2.19}$$

We see that the right-hand-side of the last equality only involves the Hilbert space $\mathcal{H}_1$. It would be desirable to find a state ρ_1 associated only with $\mathcal{H}_1$ that is able to reproduce the statistics of measurements on $\mathcal{H}_1$, given the state ρ. Indeed, this is possible by *tracing out* system 2. Define the *partial trace* over the system 2 as

$$\operatorname{tr}_2 |\psi\rangle\langle\psi| \otimes |\phi\rangle\langle\phi| = |\psi\rangle\langle\psi| \operatorname{tr}(|\phi\rangle\langle\phi|). \tag{2.20}$$

Then we can assign a state to system 1 by tracing out system 2:

$$\rho_1 = \operatorname{tr}_2 \rho; \tag{2.21}$$

ρ_1 is known as the *reduced state*. It easy to check that this produces the correct statistics for measurements on subsystem 1 only, *i.e.*,

$$\langle P_k \otimes \mathbb{I} \rangle = \operatorname{tr} P_k \rho_1. \tag{2.22}$$

By using the fact that the trace yields an inner product on the space of self-adjoint operators, one can show that ρ_1 is the only density operator that does the job for all observables.

Even though we started with the pure state (2.17), considering measurements on subsystems naturally leads to density operators and potentially mixed states. Density operators are the natural objects to study multipartite systems. Also interesting to note that the reduced state ρ_1 will in general be a mixed state even though the original state ρ is pure. This is one of the hallmarks of entanglement, as we will see in section 2.2, and will play a crucial role in the phenomenon of decoherence, section 2.3. A mixed state can reproduce the statistics of different ensembles, but it also reproduces the statistics of a system that is entangled with an unobserved system.

2.1.4 Evolution, instruments and channels

Since the density operator is a natural generalisation the state vector in the context of classical uncertainty and multipartite systems, and since quantum information theory is interested in arbitrary manipulations of a given system, it is good to study possible maps between density operators on their own terms. We proceed in an axiomatic way, and then connect with a physical interpretation. This argument is similar to that in [183].

First, let us allow that the input and output Hilbert spaces $\mathcal{H}_{\text{in}}$ and $\mathcal{H}_{\text{out}}$ are different. This way, we can represent operations such as adjoining a system, or ignoring parts of the system. Since quantum experiments yield different outcomes probabilistically, we define an evolution by a set of maps $\mathcal{E} = \{E_i\}$ from positive operators on $\mathcal{H}_{\text{in}}$ to positive operators on $\mathcal{H}_{\text{out}}$, where i labels one of the mutually exclusive outcomes. We ask that $\operatorname{tr} E_i \rho$ is the probability of the outcome i to happen and thus that

$$0 \leq \operatorname{tr} E_i \rho \leq 1 \quad \text{and} \quad \sum_i \operatorname{tr} E_i \rho = 1, \tag{2.23}$$

for all density operators ρ. Next, we ask each of the E_i to be *convex linear*, meaning that

$$E_i(p\rho + (1-p)\sigma) = pE_i\rho + (1-p)E_i\sigma \tag{2.24}$$

for any two states ρ and σ and probability p. This is so that the probabilities given by $\operatorname{tr} E_i \rho$ behave consistently with stochastic mixtures. Finally, we want the state after applying E_i to still be a positive operator. This requires that E_i is a *completely positive* map. *Positive* means that $E_i \rho$ is a positive operator on $\mathcal{H}_{\text{out}}$ whenever ρ is a positive operator on $\mathcal{H}_{\text{in}}$. *Completely positive* means that, for arbitrary $\mathcal{H}_{\text{A}}$, $(I_{\mathcal{H}_{\text{A}}} \otimes E_i)$ is a positive map from the operators on $\mathcal{H}_{\text{A}} \otimes \mathcal{H}_{\text{in}}$ to those on $\mathcal{H}_{\text{A}} \otimes \mathcal{H}_{\text{out}}$, where $I_{\mathcal{H}_{\text{A}}}$ is the identity map on operators of $\mathcal{H}_{\text{A}}$. The requirement of complete positivity ensures that the applying $\mathcal{E}$ to a subsystem always yields a well-defined state of the combined system.

In sum, each of the maps E_i is a *trace non-increasing, completely positive* (CP) map. If the outcome i attains, then the state is updated to

$$\rho \longmapsto \frac{E_i\rho}{\operatorname{tr} E_i\rho}, \tag{2.25}$$

which is again a density operator. The set $\mathcal{E} = \{E_i\}$ is called an *instrument*, or *quantum operation*. We can also define a map induced by $\mathcal{E}$ itself. If we do not know or care about the result, we weight each of the outcomes above by their probability $p(i|\rho,\mathcal{E}) = \operatorname{tr} E_i\rho$ and then we have

$$\mathcal{E}\rho = \sum_i p(i|\rho,\mathcal{E})\frac{E_i\rho}{\operatorname{tr} E_i\rho} = \sum_i E_i\rho. \tag{2.26}$$

Any evolution with a single outcome, like $\mathcal{E}$, or an instrument with a single outcome, is known as a *quantum channel*, and is represented by a *completely positive, trace preserving* (CPTP) map. The formalism of positive operators and completely positive maps is the most general way to formulate the evolution of quantum systems.

Let us connect this rather abstract formalism to unitary evolution. If a system undergoes a unitary evolution U, then its density operator changes as

$$\rho \longmapsto U[\rho] = U\rho U^\dagger, \tag{2.27}$$

which is derived by evolving each state in an ensemble that gives ρ. The map U is then known as a *unitary channel*. Suppose instead that the system under consideration interacts with another system initially in a pure state $|a\rangle$, so that the evolution of the combined system is a unitary channel, and then the second system is ignored. The resulting evolution for the state ρ of the initial system is

$$\rho \longmapsto \operatorname{tr}_2\left[U[\rho\otimes|a\rangle\langle a|]\right]. \tag{2.28}$$

This is a CPTP map. Indeed, *any* CPTP map can be represented this way, this result is known as Stinespring dilation [250]. Say that the ancillary system instead is subjected to a projective measurement represented by the projectors $\{P_k\}$, then each map

$$\rho \longmapsto \left[(I\otimes P_k)U[\rho\otimes|a\rangle\langle a|]\right]. \tag{2.29}$$

is a trace non-increasing CP map, and together they form an operation. Thus every instrument can be understood as the system interacting with an ancilla, and then the ancilla being measured. This result is known as Ozawa dilation [193].

2.2 Entanglement

It is hard to understate the importance of quantum entanglement in the field of quantum information. The use of entangled states is the key element in many of the protocols that make quantum information superior to classical information, such as secure distribution of cryptographic keys [96]; superdense coding [23], in which re-uniting two entangled qubits allows to transmit two bits of information while transmitting a single qubit; quantum state teleportation [22], allowing to transfer a full qubit state by sending two classical bits of information; and Shor's algorithm [242] for factoring large numbers.

Operationally, the property that makes entangled states special is that they allow to create correlations between spatially separated system that are stronger than what can be achieved with classical information systems. These correlations are so strong, in fact, that they put in question our very notions of agency and causality. We are referring of course to Bell's theorems[1] [16, 18], which we will discuss later in more detail, in section 3.4.

We will forego a detailed explanation of the various applications of quantum entanglement, referring the reader to standard texts, such as [137, 183]. In this section, we will focus on the most relevant part of quantum entanglement for the work in this thesis, namely, the experimental certification of entanglement and its role in the decoherence process.

The mathematical definition of entanglement is as follows. Given two systems associated with Hilbert spaces $\mathcal{H}_1$ and $\mathcal{H}_2$, respectively, the Hilbert space of the combined system is given by the tensor product $\mathcal{H}_1 \otimes \mathcal{H}_2$ of the two Hilbert spaces. A state ρ of the combined system is said to be *separable* if it can be written as a convex combination of a number of factor states, *i.e.* if

$$\rho = \sum_n p_n \; \rho_n \otimes \sigma_n. \tag{2.30}$$

A state is *entangled* if it is not separable.

2.2.1 Entanglement certification

Given the state of a system, one can check if the state is entangled by checking that it is not separable. If the full state ρ is pure, then there is fairly easy way check if the system is entangled or not. If $\rho = |\psi\rangle\langle\psi|$ then ρ is separable if and only if $|\psi\rangle$ is separable, meaning $|\psi\rangle = |a\rangle\,|b\rangle$. However, unavoidable measurement noise and interactions with the environment will cause most systems to display mixed state statistics. Thus we need criteria to detect separability in mixed states. In general, for an arbitrary tensor decomposition of an arbitrary Hilbert space, proving that a given state is separable is quite hard, approaching NP-hardness, but there are a number of both necessary and sufficient conditions for separability (see [159] and citations therein).

An easy to check, sufficient condition for separability is known as the *positive partial transpose* criterion. The partial transpose ρ^Γ of a bipartite state ρ is obtained by transposing on only one of the subspaces:

$$\langle im|\rho^\Gamma|jn\rangle = \langle in|\rho|jm\rangle\,. \tag{2.31}$$

If ρ^Γ is a positive operator, then ρ is separable [201]. For states of $\mathbb{C}^2 \otimes \mathbb{C}^2$ and $\mathbb{C}^2 \otimes \mathbb{C}^3$, this condition is both necessary and sufficient [136]. This is all that would

[1]While not every entangled state allows to violate the Bell inequalities on their own, for any state ρ that does not violate the inequalities, there is another state σ that also does not violate the inequalities, such that $\rho \otimes \sigma$ allows the violation [170]. This means that any entangled state ρ displays qualities that cannot be replicated by classically correlated systems.

be needed in practice to verify or falsify the presence of entanglement in the GME experiment discussed in section 1.4.1.

Knowledge of the full quantum state can only be obtained via quantum state tomography. In quantum state tomography, one measures the expectation values of a set of different observables that are sufficient to fix all the independent components of the quantum state. This process can be quite costly, as it requires to perform the experiment repeatedly to estimate with enough precision all of the observables.

Consider for example the case of two qubits. A positive operator on $\mathbb{C}^2 \otimes \mathbb{C}^2$ has 16 complex components. Self-adjointedness implies that the four diagonal components are real and of the remaining 12, only 6 are free to take any complex value. The trace 1 requirement imposes one further real constraint, so there are in total $4 + 6 \times 2 - 1 = 15$ free real components. These can be obtained by measuring $\langle \sigma_\mu \otimes \sigma_\nu \rangle$, $\mu, \nu = 0, 1, 2, 3$, where σ_0 is the identity, and σ_i are the Pauli matrices. In practice, this requires only 9 different measurement schemes, one for each correlation $\langle \sigma_i \otimes \sigma_j \rangle$, as the remaining 6 single-system expectation values can be obtained from the resulting data set. In general, for N qubits, there are $4^N - 1$ real components, that can be estimated with 3^N measurements schemes. The fact that the state can be inferred by performing measurements on each system separately, a property known as *tomographic locality*, plays a key role in the reconstructions of quantum mechanics, see section 3.1.

Fortunately, there also exist techniques that provide a sufficient condition to detect the presence of entanglement without knowing the quantum state. One such condition is of course the violation of a Bell inequality. But not all entangled states allow the violation of a Bell inequality on their own. A more practical technique is that of detection of entanglement via an *entanglement witness* [136, 252]. An entanglement witness W is an observable such that

$$\operatorname{tr} W\sigma \geq 0 \tag{2.32}$$

for all separable σ but

$$\operatorname{tr} W\rho_0 < 0 \tag{2.33}$$

for at least one entangled state ρ_0. It follows that any state such that $\operatorname{tr} W\rho > 0$ is an entangled state. Thus, measuring a negative expectation value for W in the lab implies that the state generated is an entangled state. The intuition behind this is pretty simple. We recall the map

$$(A, B) \longmapsto \operatorname{tr} AB \tag{2.34}$$

is a scalar product on the space of Hermitian operators, making the latter a normed real vector space. The set of S separable density operators is a convex subset of this space. It follows that for any point ρ_0 not in S, there is a hyperplane separating S from ρ_0. Then every point on the same side of the hyperplane as ρ_0 is automatically not in S.

Note an important difference between an entanglement witness and the violation of a Bell inequality: the latter is relatively theory-independent. To claim that one has seen a negative value for $\langle W \rangle$ implies one is confident in the workings of one's apparata and knows what observables are being measured. This normally entails assumptions about the quantum physics of the experimental setup. In contrast, Bell's theorem imposes constraints on correlations between a set of observables, regardless of what these observables are, thus, a violation of a Bell inequality is said to be a *device independent* way of certifying entanglement.

2.2.2 Application to the GME

Having an idea of the state of your system helps to build the entanglement witness W to certify the presence of entanglement. As we saw in section 1.4.1, quantum gravity predicts that the state of the spins at the end of the experiment is

$$|\psi\rangle_{\mathrm{M}} = |0\rangle \left(e^{i\tilde{\phi}_{LR}} |0\rangle + |1\rangle\right) + |1\rangle \left(|0\rangle + e^{i\tilde{\phi}_{RL}} |1\rangle\right), \tag{2.35}$$

where we are using the computational basis $|0\rangle = |\uparrow\rangle, |1\rangle = |\downarrow\rangle$ In practice, the actual state might differ from the one above because there will be noise in the implementation of the experiment, but the actual state should approximate this one. Bose *et.al.* [34] proposed this entanglement witness

$$W_{\mathrm{B}} = I \otimes I - \sigma_x \otimes \sigma_z - \sigma_y \otimes \sigma_y, \tag{2.36}$$

which evaluates to

$$\langle W_{\mathrm{B}} \rangle = \frac{1}{2}\left[1 + \cos\!\left(\tilde{\phi}_{RL} - \tilde{\phi}_{LR}\right) + \cos\tilde{\phi}_{RL} - \cos\tilde{\phi}_{LR}\right], \tag{2.37}$$

where we recall that $\tilde{\phi}_{RL}$ and $\tilde{\phi}_{LR}$ are proportional and grow linearly in time, with $\tilde{\phi}_{RL} > \tilde{\phi}_{LR}$. While $\langle W_{\mathrm{B}} \rangle$ will eventually become negative, Chevalier *et.al.* [54] pointed out that the witness

$$W_{\mathrm{C}} = I \otimes I - \sigma_x \otimes \sigma_x + \sigma_x \otimes \sigma_y + \sigma_y \otimes \sigma_z, \tag{2.38}$$

which gives

$$\langle W_{\mathrm{C}} \rangle = \frac{1}{2}\left[1 - \cos\!\left(\tilde{\phi}_{LR} - \tilde{\phi}_{RL}\right) - \sin\tilde{\phi}_{LR} - \sin\tilde{\phi}_{RL} - \sin\!\left(\tilde{\phi}_{LR} - \tilde{\phi}_{RL}\right)\right], \tag{2.39}$$

is negative for arbitrarily small values of free fall time, and is thus better suited to the experiment.

2.2.3 Local operations, classical communication

A well-known result in quantum information theory is that two spatially separated agents, each acting on their own quantum system, cannot create entanglement between their systems, even if they coordinate their actions by transmission of classical information [137].

The argument is quite simple. A *local operation* is represented by a map $\mathcal{E} \otimes \mathcal{F}$, where $\mathcal{E}$ and $\mathcal{F}$ are channels. The channels could represent an instrument where no post-selection is made on the basis of the result; this is because the two parties are not allowed to communicate, so they cannot coordinate their behaviour. Allowing one transmission of classical information from one party to the other, allows the choice of one operation conditional on the result of the other. This situation is one round of LOCC (local operations and classical communications) and is represented[2] by a map

$$\sum_i E_i \otimes \mathcal{F}^{(i)}, \tag{2.40}$$

where $\{E_i\}$ is an instrument and each $\mathcal{F}^{(i)}$ is a channel. The $\mathcal{F}^{(i)}$s have to be deterministic channels as the second party is not allowed to send the result of their operation to the first party in this scenario. The map representing a round of LOCC where the second party sends information to the first is defined similarly. If the state of the system is initially separable, it will still be separable after one round of LOCC, since

$$\sum_i \left(E_i \otimes \mathcal{F}^{(i)}\right)(\rho \otimes \sigma) = \sum_i E_i\rho \otimes \mathcal{F}^{(i)}\sigma = \sum_i p_i\, \rho_i \otimes \sigma_i. \tag{2.41}$$

Thus an initially separable state will remain separable after any number of LOCC rounds.

This result was used in [34] to argue that the detection of GME would prove that gravity is a quantum force. However, as pointed out by [164], one might distrust this argument based completely on quantum theory, as we do not know that gravity obeys either quantum or classical laws, and might follow a new set of laws. They argue that one ideally needs a similar argument in a more general framework. This is achieved in [165] and [112], as explained in sections 2.4 and 2.5.

2.3 Decoherence

Decoherence is the phenomenon by which quantum interference effects are cancelled because of entanglement. As we will see, a certain amount of decoherence is inevitable in most realistic setups, as the environment will get entangled with the system under consideration. This suppresses interference effects in systems that have undergone large spatial superpositions. Decoherence is one of the main obstacles in performing quantum experiments. On the other hand, decoherence is one of the main mechanisms that allows the classical world to emerge from the quantum world, as we will see in chapter 8.

2.3.1 In the double slit experiment

Decoherence is most easily demonstrated by considering the double-slit experiment with photons.

Suppose the state of the photons at the screen is given by

$$|\psi\rangle = \frac{1}{\sqrt{2}}\,|\psi_A\rangle + \frac{1}{\sqrt{2}}\,|\psi_B\rangle\,, \tag{2.42}$$

where $|\psi_A\rangle$ is the what state would be if only slit A were open and $|\psi_B\rangle$ the one with only B open. The density of the detection events on the screen, as a function

[2] This argument can easily be extended to model the transmission of information by an actual physical system. We will not do this here, since we will overview a more general argument in section 2.5.

of the position z, is given by

$$I_\psi(z) \propto |\langle z|\psi\rangle|^2 = \frac{1}{2}|\langle z|\psi_A\rangle + \langle z|\psi_B\rangle|^2. \tag{2.43}$$

The resulting pattern is not proportional to the average of the intensities resulting from having either one or the other slit open:

$$I_\psi(z) \neq I_A(z) + I_B(z) \propto |\langle z|\psi_A\rangle|^2 + |\langle z|\psi_B\rangle|^2. \tag{2.44}$$

Notably, I_ψ has dark bands, known as interference fringes, where $I_A + I_B$ has none.

Now let us include a nearby atom in the description, which could be in either of two states $|0\rangle$ and $|1\rangle$. Let us assume that the state of the whole system is then

$$|\Psi\rangle|0\rangle = \frac{1}{\sqrt{2}}|\psi_A\rangle|0\rangle + \frac{1}{\sqrt{2}}|\psi_B\rangle|0\rangle. \tag{2.45}$$

Of course, the resulting pattern on the screen is the same as in (2.43). However the formula is obtained by tracing away the atom

$$I_\Psi(z) \propto \langle z|(\mathrm{tr}_{\mathrm{atom}}\,|\Psi\rangle\langle\Psi|)|z\rangle \tag{2.46}$$

where, by definition,

$$\mathrm{tr}_{\mathrm{atom}}(|\Psi\rangle\langle\Psi|) = \langle 0|\Psi\rangle\langle\Psi|0\rangle + \langle 1|\Psi\rangle\langle\Psi|1\rangle = |\psi\rangle\langle\psi|, \tag{2.47}$$

so that, indeed $I_\Psi(z) = I_\psi(z)$.

Consider now a modification of the experiment where, whenever a photon [104] is sent through the slits, a new atom in the state $|0\rangle$ is placed in the middle of slit B. Suppose that the energy of the photon is such that it causes a transition $|0\rangle \mapsto |1\rangle$ when it interacts with the atom. Thus the state of the photon and atom system is

$$|\Phi\rangle = \frac{1}{\sqrt{2}}|\psi_A\rangle|0\rangle + \frac{1}{\sqrt{2}}|\psi_B\rangle|1\rangle. \tag{2.48}$$

The atom is now entangled with the path of the photon, and the resulting slit pattern on the screen will be different. Indeed,

$$\mathrm{tr}_{\mathrm{atom}}\,|\Phi\rangle\langle\Phi| = \frac{1}{2}|\psi_A\rangle\langle\psi_A| + \frac{1}{2}|\psi_B\rangle\langle\psi_B|, \tag{2.49}$$

so that the intensity on the screen will be

$$I_\Phi(z) \propto |\langle z|\psi_A\rangle|^2 + \langle z|\psi_B\rangle|^2 \propto I_A(z) + I_B(z). \tag{2.50}$$

Even though each photon is still in a superposition of going through either slit, the interference fringes disappear. The entanglement between the path of the photon and the state of the atom, together with the fact that the atom is not measured by the screen, leads to the disappearance of interference effects. This allows to reason *as if* each photon really went through one slit *or* the other.

Decoherence preserves the self-consistency of quantum mechanics. The photon acts *as if* its position has been detected at one or the other slits, and this is necessary, because one could decide to measure the atom to learn where the photon has been. One could measure the atom any time after the interaction, or the atom itself could be part of the measuring apparatus. This is seen as an example of the possibility of consistently moving the Heisenberg cut between the quantum description and the classical one.

Let us consider a slightly different scenario, in which the photon leaves the atom in the state $|\alpha\rangle = \cos\alpha\,|0\rangle + \sin\alpha\,|1\rangle$, resulting in the state

$$|\Phi_\alpha\rangle = \frac{1}{\sqrt{2}}\,|\psi_A\rangle\,|0\rangle + \frac{1}{\sqrt{2}}\,|\psi_B\rangle\,|\alpha\rangle\,. \tag{2.51}$$

Then,

$$\mathrm{tr}_{\mathrm{atom}}\,|\Phi_\alpha\rangle\!\langle\Phi_\alpha| = \frac{1}{2}\,|\psi_A\rangle\!\langle\psi_A| + \frac{1}{2}\,|\psi_B\rangle\!\langle\psi_B| + \frac{1}{2}\cos(\alpha)\big(\,|\psi_A\rangle\!\langle\psi_B| + |\psi_B\rangle\!\langle\psi_A|\,\big). \tag{2.52}$$

Thus we see that, as α changes from 0 to $\pi/2$, the off-diagonal entries in the reduced state for the path gradually disappear. The interference effect is reduced only to the extent that a measurement of the atom can give reliable which-way information about the path of the photon. In this way, decoherence can be seen as a consequence of information about a system leaking out into the world.

2.3.2 Inevitable environmental decoherence

In a real experiment, it is impossible to completely isolate the system under study from the rest of the world. Since the experimental apparatus is at finite temperature, there will be a thermal photon bath in the cavity, and since it is impossible to make a perfect vacuum, there will always be some probability that the system collides with some stray gas molecule. These interactions will lead to entanglement between the system and the environment and lead to decoherence.

Say the system and environment are initially in the product state

$$|\Psi_{t_0}\rangle = \frac{1}{\sqrt{2}}\big(\,|0\rangle + |1\rangle\,\big)\,|\psi\rangle\,, \tag{2.53}$$

where $|0\rangle$ and $|1\rangle$ denote two different semi-classical states centred around two different locations. The environment is affected differently depending on the position of the particle and, at some later time t_1, the total system is in the state

$$|\Psi_{t_1}\rangle = \frac{1}{\sqrt{2}}\,|0\rangle\,|\psi_0\rangle + \frac{1}{\sqrt{2}}\,|1\rangle\,|\psi_1\rangle\,. \tag{2.54}$$

Experiments on the particle alone will be ruled by the reduced state

$$\rho = \mathrm{tr}_{\mathcal{E}}\,|\Psi_{t_1}\rangle\!\langle\Psi_{t_1}| = \frac{1}{2}\big(\,|0\rangle\!\langle 0| + |1\rangle\!\langle 1| + \epsilon\,|0\rangle\!\langle 1| + \epsilon^*\,|1\rangle\!\langle 0|\,\big), \tag{2.55}$$

where $\epsilon = \langle\psi_1|\psi_0\rangle$. The time-evolution of the reduced state ρ can be modelled in various situations with the help of the *master equation* [218]:

$$\frac{d}{dt}\,\langle x|\rho|x'\rangle = \frac{i}{\hbar}\,\langle x|\,[\rho, H]\,|x'\rangle - \Gamma(x - x')\,\langle x|\rho|x'\rangle\,, \tag{2.56}$$

where the first term is just the Schrödinger evolution induced by the dynamics of the particle alone, and the second term causes the off-diagonal terms to decay exponentially in time. The function Γ determines the decoherence rate, and can be modelled by

$$\Gamma(\Delta x) = \gamma \left(1 - \exp\left[-\left(\frac{\Delta x}{2a}\right)^2\right]\right), \tag{2.57}$$

were $\gamma > 0$ is a constant *localisation strength* and $a > 0$ is a *localisation length.* The precise values for these parameters have to be derived by modelling the interactions of the particle with the particular environment. There are two different regimes, known as long-wavelength (LW) and short-wavelength (SW) regimes, depending on wether $\Delta x \ll 2a$ or $\Delta x \gg 2a$. In the LW regime we have

$$\Gamma_{\mathrm{LW}}(\Delta x) \approx \gamma \frac{(\Delta x)^2}{4a^2} \equiv \Lambda(\Delta x)^2, \tag{2.58}$$

so that decay rate is proportional to the square of the distance in the superposition, with some proportionality factor Λ. In contrast, in the SW regime, the decoherence rate reaches a maximum

$$\Gamma_{\mathrm{SW}}(\Delta x) \approx \gamma. \tag{2.59}$$

This corresponds to regimes where a single interaction can completely localise the particle.

In the experiments considered in this thesis, interactions with thermal photons will lead to LW decoherence, as the average wavelength of a photon at 5 K is about 1 mm, which is much larger than the superposition sizes considered. Interactions with gas molecules, on the other hand, will be in the SW regime, as the typical de Broglie wavelength of a gas molecule will be smaller than the superposition size.

2.4 General probabilistic theories

We now broaden our horizons to consider a framework that generalises the operational formulation of quantum theory. The main idea behind operationalism is that any scientific theory should—at the very least—provide probabilistic predictions about laboratory procedures. One might argue that a good theory scientific theory should also provide an explanation, or a picture of what is going on in nature, and tell us something about the world outside our laboratories, but a theory cannot be a good scientific theory if it can't tell us what happens in a given experiment.

This idea has led to the development of a few related mathematical frameworks, most notably Generalised Probabilistic Theories (GPTs) [13], Operational Probabilistic Theories (OPTs) [73], and Process Theories [68], and, more recently, Constructor Theory [163]. These are frameworks in which the predictive content of different physical theories can be formulated and compared. They are of great use in studying the information-processing capabilities of different theories, in a similar way that, say, linear algebra is useful in studying the formal properties of physical theories as different as quantum mechanics and fluid dynamics: knowing that your theory is a OPT with such and such properties allows you to immediately derive a host of results. For example, as we will shortly see, in every GPT it is impossible to send a signal without exchanging a system.

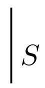
These frameworks have been put to fruitful use in the programme of the reconstruction of quantum mechanics, as we will see in section 3.1. Here instead, we will provide a brief introduction to GPTs, which we will use in the next section for a proof of the no-go theorem by Galley, Giacomini, and Selby [112] about the non-classical nature of the gravitational field.

2.4.1 States, transformations and effects

Mathematically, a GPT system S consists of a convex set of states Σ_S, a convex set of transformations T_S and a convex set of effects E_S. In general there will be a vector space V_S associated with S, such that the states correspond to vectors, effects to dual vectors and transformations to linear maps $V_S \to V_S$. States, effects and transformations are called *processes*.

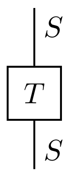
Operationally, the states Σ_S correspond to equivalence classes of preparations of the system, the transformations T_S correspond manipulations (including observations) that can be done to the system without destroying it or losing it and effects E_S represent manipulations (including observations) after which the system is then destroyed or simply ignored.

GPTs come with an expressive diagrammatic calculus in which systems are represented by (labelled) wires



(2.60)

and processes are represented by boxes with dangling wires

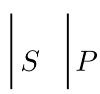


. (2.61)

In particular, states have no input wires and effects have no output wires:

. (2.62)

The diagrammatic calculus becomes particularly useful when one starts considering different systems and how they interact. Two GPT systems can be composed in parallel by using the tensor product structure of the associated spaces. The resulting system is then represented by wires side by side

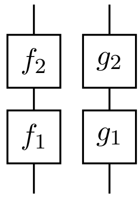


. (2.63)

The multi-system diagrammatic calculus provides two related advantages over traditional "1D" formulas. First, a formal advantage, since it makes redundant a number of equations relating parallel and sequential composition. For example, the property

$$(f_2 \circ f_1) \otimes (g_2 \circ g_1) = (f_2 \otimes g_2) \circ (f_1 \otimes g_1) \tag{2.64}$$

becomes self-evident, as when building the diagrams corresponding to the left hand side and the right hand side of the formula above, one obtains the same diagram, namely,

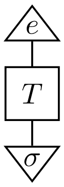


. (2.65)

The second advantage is one of readability: when there are many systems interacting, it can start to become difficult to read the formulas, while the diagrams remain lucid.

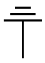
2.4.2 Probabilistic interpretation

A diagram with no free dangling wires such as



(2.66)

corresponds to a scalar (a state in the trivial GPT system). By adding extra conditions on a GPT, one can interpret every scalar as a probability. In some GPTs, there is a distinguished effect, called the *discard*, represented as

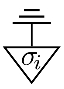


, (2.67)

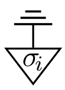
which represents ignoring the system from that point onwards. This allows to define probabilities in the following way. Let us introduce the empty diagram

(2.68)

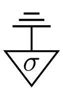
to represent the number 1. Then a set of states $\{\sigma_i\}$ such that

$\sum_i$  =  (2.69)

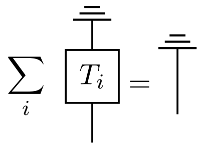
represents a preparation with a probabilistic outcome, where the label i serves to identify the different possible outcomes. Then the probability of outcome i happening is given by

$P\left(i|\{\sigma_i\}\right) =$  . (2.70)

A *normalised* state, is one such that

 =  . (2.71)

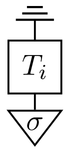
Similarly, an *operation*, also known as an *instrument* is a set of transformations $\{T_i\}$ such that



(2.72)

and the probability of the particular transformation T_i happening, given that the system was in the state σ is

$$P\left(i|\{T_i\},\rho\right) =$$



(2.73)

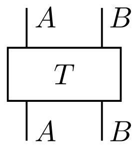
The requirement of convexity is now understood as the requirement that, given the experimental ability of preparing the state σ and the ability of preparing the state σ', one is also able to flip a biased coin and prepare the state σ with probability p or σ' with probability $1-p$. This preparation then corresponds to the state

$$p\sigma + (1-p)\sigma'. \tag{2.74}$$

2.4.3 Causality and the conservation of probabilities

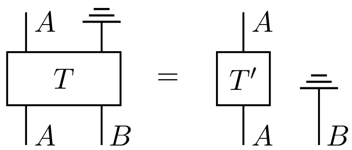
As we have just seen, the discard is closely related to the idea that probabilities always sum to 1. However, there is a close relation in GPTs between the conservation of probabilities and notions of *causality.* Indeed, diagrams that only make use of deterministic processes to propagate information, two processes can affect each other only if there is a system connecting them. Moreover, the result of an operation cannot be affected by the nature of a *later* operation. Let us see why.

A transformation T' on two systems A and B



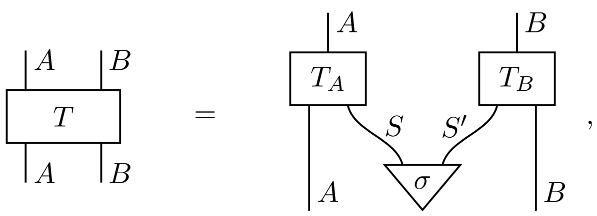
(2.75)

is said to be *non-signalling from* A *to* B if



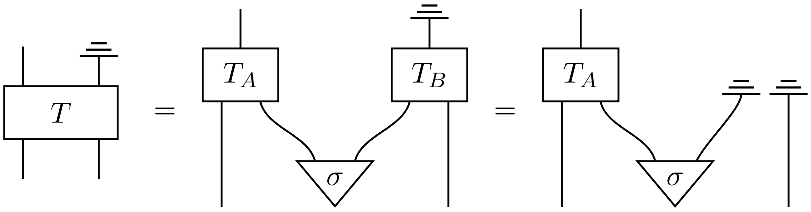
(2.76)

for some T', meaning that if one ignores B after applying T, then one can compute the resulting transformation on A without knowing the initial state of B. Put in other words, the initial state of B does not affect the statistics of A via T. It is immediate to prove that any transformation of the form:



(2.77)

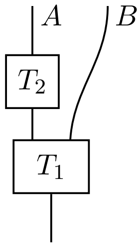
where T_A, T_B and σ are normalised, is non-signalling from A to B and from B to A. Indeed



(2.78)

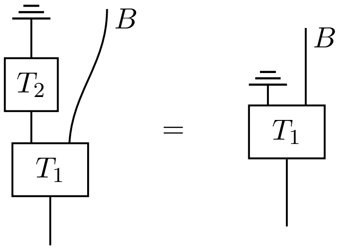
Note that this is quite a general statement. Indeed, σ could be a quantum entangled state, or a state of a theory that allows even stronger correlations. The only substantial requirement is that S and S' are GPT systems.

Similarly, consider a diagram of the form



(2.79)

where both T_1 and T_2 are normalised. Then to compute the effect on system B, there is no need to know anything about T_2, since



(2.80)

Thus, in GPTs, later operations do not affect the outcomes of earlier operations. This property is also called *no-signalling from the future.*

There is a stark time-directionality in this formalism. GPTs are operational theories, designed for reasoning about what happens in the lab. In most cases, one is interested in *predicting* the results of the experiment, given the setups. Thus all probabilities computed with GPTs are understood as prediction probabilities about a set of events, given knowledge about events in their past. We will have more to say about this in chapter 7.

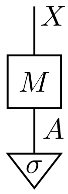
2.4.4 Classical and non-classical systems

Quantum theory is a paradigmatic example of a GPT theory, with states as density matrices, CPTP maps as transformations and taking the trace as the unique effect.

A suitable GPT model of a finite-dimensional classical system can be constructed as follows. Let X be the finite set of configurations of the system. Then the states of the GPT are the probability distributions over X, the transformations are stochastic maps on these distributions, and the only effect (the discard) is marginalisation. The state space can be embedded in a $|X|$-dimensional real vector space V_X, the stochastic transformations are then represented by stochastic matrices and the discard amounts to summing all entries in the vector. Classical systems may be combined by making use of the tensor product of the underlying vector spaces.

The simplest nontrivial example of a classical system is the classical bit. It is associated with the space $X = \{0, 1\}$ and its state space is isomorphic to the line segment $[0, 1]$. The simplest nontrivial quantum system is the qubit, whose state space is isomorphic to a ball in 3D space.

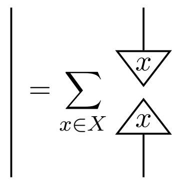
Classical systems can be used to model measurements on non-classical systems. Let M be a convex map from the states of A to states of a classical GPT system X, and let σ be a state for system A, then



(2.81)

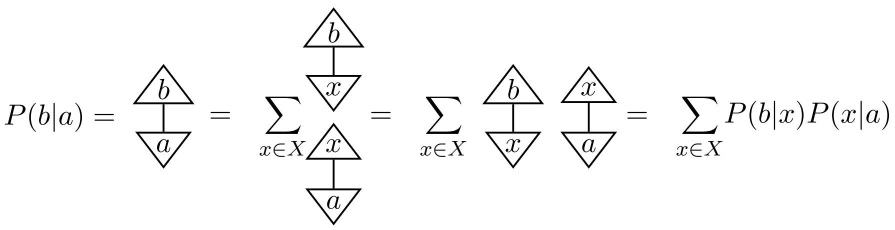
is a probability distribution over X, which can represent the probability of various outcomes of the measurement, as read on a classical pointer variable.

We can diagrammatically express what makes a system classical or not. For example, all classical systems have a crucial property, namely, that the identity operation can be decomposed as a sum of (or integral over) projectors:



(2.82)

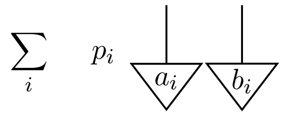
Thus, when computing probabilities about classical systems, one can use the classical probability axiom



(2.83)

This can be taken as meaning that a classical system is always in one of its states, and all probabilistic considerations are a result of ignorance. Or it can be taken to say that classical systems can be measured without perturbing the state. Or yet again, there are no interference effects in classical systems. Either way, this property does not hold in a general GPT, and in particular, it does not hold in quantum theory.

Another property shared by all classical systems is that of *state separability.* A bipartite state is *separable* if it can be written as a convex combination of factored states:



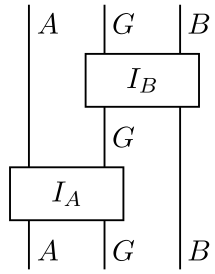
(2.84)

All bipartite classical systems have separable states. This follows easily from the defining property (2.82). Thus, if a GPT system has non-separable states, then the system cannot be a classical system.

2.5 A no-go theorem about the gravitational field

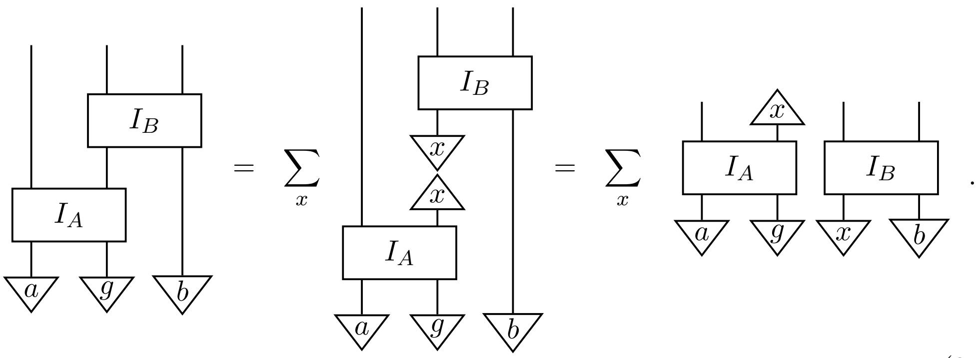
We have now set up the machinery needed to understand the proof of the no-go theorem by Galley, Giacomini, and Selby [112]. This theorem states that two systems can never get entangled as a result of interacting via a classical system. It implies that a positive result of the GME experiments is proof that the gravitational field is not a classical system.

More specifically, assume that we have three systems A, B, and G, such that G is a classical and that A and B interact only with G. That is, we can write the evolution of the system as:



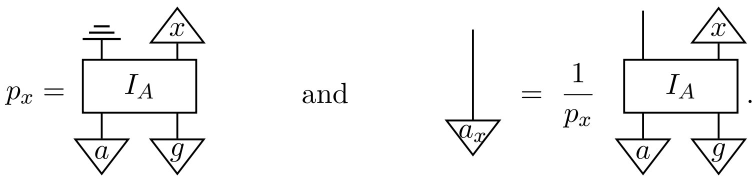
(2.85)

or as a sequence of such interactions. Then A and B cannot become entangled. Indeed, assume that the three systems start in a separable state and use the defining property (2.82) of a classical system to write



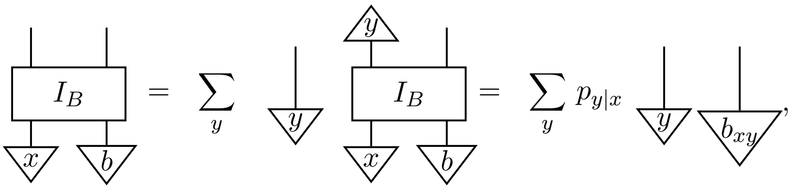
(2.86)

Then define



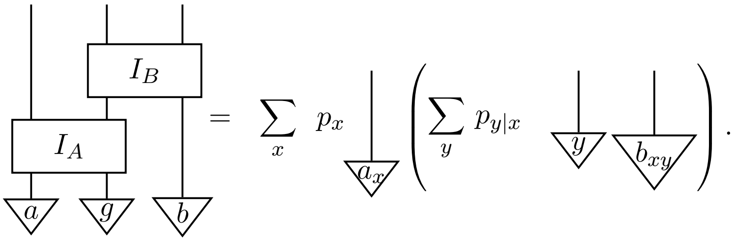
(2.87)

Since I_A, a and g are all normalised, we have $\sum_x p_x = 1$, and thus the a_x are also normalised states. Thus (2.86) already shows that A is not entangled with B and G. To show that B and G are not entangled, we use again the defining property (2.82) of classicality



(2.88)

where we have defined the probability distribution $p_{y|x}$ and the normalised states b_{xy} in a manner analogous p_x and a_x. Putting all together,



(2.89)

The equation above shows clearly that the result of such interactions mediated by the classical system G leads to correlations, but not entanglement between A and B, as the final state is separable. This will be true of a sequence of such interactions. This completes the proof.

Thus, if we assume that the gravitational field can be represented by a GPT system (meaning broadly that it has states and observables) and that two distant systems only interact via the gravitational field, the observation of entanglement in the GME experiments means that the gravitational field is showcasing some non-classical behaviour. In particular, during the experiment it must be in a state that does not obey (2.82), which is equivalent to the notion of having non-commuting observables.

Thus, low energy quantum gravity experiments, aided by results of quantum information theory, might afford us the first glimpse of the quantum nature of gravity.

Chapter 3

Quantum Foundations

Science offers the boldest metaphysics of the age. It is a thoroughly human construct, driven by the faith that if we dream, press to discover, explain, and dream again, thereby plunging repeatedly into new terrain, the world will somehow come clearer and we will grasp the true strangeness of the universe. And the strangeness will all prove to be connected, and make sense.

Edward O. Wilson [263]

We have seen how the study of quantum information theory has led to the development of a number of tools that can be put to use in quantum gravity research. It also stimulated a systematic study of the most counter-intuitive properties of quantum systems. This is the objective of the field of quantum foundations: trying to find the answers to Wheeler's [261] question "*How come the quantum?*"

In this chapter, we review the reconstruction programme of quantum mechanics, which seeks to re-derive the mathematical structure of the theory from physical and operational axioms; we will look at the project of interpretations of quantum mechanics, that seeks to build a coherent picture of the unobserved world; and finally we will talk about the field of experimental metaphysics, a quantitative study of how much quantum mechanics defies intuitive physical principles.

3.1 The idea of a reconstruction

When first presented with the mathematical structure of quantum theory, students often think that it is rather abstract and seemingly arbitrary. It is not quite clear *why* systems have to be represented by rays in Hilbert space, observables by hermitian operators, and why the probabilities of various observations have to be computed using the Born rule. We learn the rules and how to apply them, and so we might forget how strange and arbitrary they appeared at first.

Compare the situation with special relativity. Physicists eventually realised that Maxwell's equations were not invariant under Galilean transformations

$$\begin{pmatrix} t \\ x \end{pmatrix} \longrightarrow \begin{pmatrix} t \\ x + vt \end{pmatrix} \tag{3.1}$$

but they were invariant under the Lorentz transformations

$$\begin{pmatrix} t \\ x \end{pmatrix} \longrightarrow \frac{1}{\sqrt{1 - v^2/c^2}} \begin{pmatrix} t + xv/c^2 \\ x + vt \end{pmatrix}. \tag{3.2}$$

This formula, on its own, also appears quite arbitrary. Part of the success and beauty of the theory of special relativity is that it manages to derive the formula for the Lorentz transformation purely physical postulates. Namely,

1. There is no such thing as absolute velocity.
2. The speed of light is the same in every inertial frame of reference.

Although these postulates might be counterintuitive at first sight, they are comprehensible, and they are also verified by observation. They also seem dangerously close to contradiction. Fitting them together in a coherent mathematical framework resolves this tension and recovers the Lorentz transformation. The postulates illuminate the transformation thanks to their physical nature.

Can the same be done for quantum theory? Is there a set of physically-inspired postulates that allows to derive the Hilbert space structure and helps to understand its physical significance? This are precisely the questions raised by Rovelli in 1996 [221], where he proposed a number of such postulates. But it wasn't until 2001, when Fuchs pleaded the community of quantum information theory [105], that the first reconstruction was successfully achieved by Hardy [124]. Hardy's reconstruction has subsequently been refined, and a number of other original reconstructions have appeared since [57, 74, 125, 132, 133, 146, 171, 187, 239]. In many reconstructions, quantum theory differs from classical theory by a single axiom, which then formally identifies what makes quantum special. Below, we will review two examples of differing approaches.

Besides their mathematical and philosophical interest, reformulating quantum theory in terms of physical postulates has another advantage: it suggests ways of modifying quantum theory by relaxing or replacing one of the postulates. Thus they could prove fruitful in the development of a new theory, if the task of finding a theory of quantum gravity proves impossible. In this connection, we note that there is a common feature of all reconstructions (with the notable exception of Jia's [146]): they are all time-oriented. So far, all fundamental theories of mechanics have been time-symmetric, and it is possible that quantum gravity, too, will have its notion of time-reversal symmetry. In chapter 7, we discuss the origin of this "operational" arrow of time, and argue that it is not a necessary feature of quantum theory.

3.1.1 Is entanglement special?

The reconstruction by Masanes and Müller [171] is a refinement of Dakić and Brukner's [74], which is itself a refinement of Hardy's original reconstruction [124].

It is based on the GPT framework discussed in section 2.4 and thus assumes the concepts of states, effects and transformations as primitives and seeks to fix the mathematical structure of quantum theory from the properties of systems in these prepare-and-measure scenarios.

The *information carrying capacity* of a system is the maximal number of perfectly distinguishable states. A set of states $\{s_1, \ldots, s_n\}$ are *perfectly distinguishable* if there is a measurement X with outcomes $\{x_m\}$ such that $p(x_m|s_n) = \delta_{mn}$. A *pure* state is a state that cannot be expressed as the convex combination of any other two states. The postulates then are:

1. **Finiteness.** If a system carries one bit of information, then each state is characterised by the outcome probabilities of a finite set of measurements.
2. **Local tomography.** The state of a composite system is fully characterised by the statistics of measurements performed on the subsystems.
3. **Equivalence of subspaces.** Systems that carry the same amount of information have isomorphic state spaces.
4. **Symmetry.** Any pure state can be reversibly transformed into any other pure state.
5. **All measurements are allowed.** Every mathematically well defined effect on a system carrying one bit corresponds to a possible measurement.

Some comments. The axioms 3, and 4 are crucial aspects of information. They codify *fungibility* of information: a bit is a bit no matter what the physical embodiment, and bits can be flipped, it does not matter which states we call 1 and which state we call 0. Axioms 1 and 2 have to do with state estimation. Finiteness says that state estimation is possible in practice, while local tomography is a form of locality: no matter how strong the correlations between the different parts of a system are, complete information about the whole system can still be recovered by probing the parts separately and then bringing the results together.

Classical probability theory and quantum theory are the only two GPTs that satisfy these 5 axioms. The authors remark that since classical probability theory is embedded in quantum theory, quantum theory is the most general GPT that satisfies the axioms. To single out quantum theory from the classical theory, one only needs to modify the requirement 4, by asking that the transformation be *continuous*.

It might appear striking that what distinguishes quantum from classical is that quantum requires a notion of *continuity* in its state space. This requirement can be understood by thinking of the paradigmatic qubits: the polarisation of a photon or the spin of an electron. These states are frame-dependent (observers in two different frames will assign a different states to these systems) and different frames are related by continuous transformations. The state of the paradigmatic classical bit is instead frame-independent.

3.1.2 Information acquisition

The reconstruction by Höhn and Wever [132, 133] focuses on how information about a system can evolve while interacting with it. Specifically, they imagine a system O interacting repeatedly with another system S, where O is capable of storing and processing information about its interactions with S. They also assume that O has developed a theoretical model $(\mathcal{Q}, \Sigma, \mathcal{T})$ for its interactions with S. The set $\mathcal{Q} = \{Q_i\}$ is a catalogue of *questions* that it can ask S (interactions with discrete outcomes), Σ is a set of states, or *catalogues of knowledge*, about S, each assigning a probability y_i for the question Q_i to yield answer 'yes'. $\mathcal{T}$ is a set of possible evolutions for the states.

They then assume that O uses broadly Bayesian methodology to update the state assigned to S, in light of the answers received from previous interactions with S. It is assumed that there is a state of no knowledge, where $y_i = 1/2$ for all questions. Two questions Q_i and Q_j are deemed *independent* if asking Q_i does not change y_j, they are deemed *complementary* if $y_i = 1$ automatically implies $y_j = 1/2$. One can then define an information measure I such that, given a maximal set of independent questions $\{Q_i\}_{i=1}^{D}$:

$$\begin{aligned} I : \Sigma &\longrightarrow [0, +\infty[\\ \vec{y} &\longmapsto \sum_{i=1}^{D} \alpha(y_i) \end{aligned} \tag{3.3}$$

where the numbers $\alpha(y_i) \in [0, 1]$ quantify the information about every single question:

$$\begin{aligned} \alpha(0) = 1 &= \alpha(1), \\ \alpha(1/2) &= 0. \end{aligned} \tag{3.4}$$

The precise form of α is determined by the axioms. They also assume the existence of an *elementary system*, one for which $I(\vec{y}) \leq 1$.

By placing restrictions on the possible form of O's model about a system S_N composed of N elementary systems, one can recover quantum mechanics of N qubits. The restrictions are:

1. **Limited information** It is possible for O to acquire up to N independent bits of information about S_N.
2. **Complementarity** It is always possible for O to obtain N *new* independent bits of information about S_N.
3. **Information preservation** The total information O has about is constant when S_N does not interact with anything.
4. **Time evolution** O's catalogue of knowledge about S_N evolves continuously with time between interrogations and every well defined evolution is possible in practice.
5. **Question unrestrictedness** Every mathematically well-defined question can be asked by O.
6. **Tomographic locality** O can determine the state of a composite system by interrogating the N components separately.

Requirements 1 and 2 are quantitative versions of the postulates 1 and 2 initially proposed by Rovelli in [221] and later by Zeilinger and Brukner [39, 271]. These are the main difference between this reconstruction and the one above. In this reconstruction, axioms 1-5 narrow down O's theory to two alternatives: real and complex quantum theory. Real quantum theory is embedded in complex quantum theory by limiting measures of qubits on two complementary bases instead of 3. Axiom 6 serves to distinguish from the two.

3.2 The measurement problem

Thus far, reconstructions focussed on the relation one observer (or experimenter) has with a quantum system. However, problems arise when one tries to mesh the description of two observers interacting with a quantum system. The Wigner's Friend thought experiment [262], in which two different people seem to give a different account of a series of physical events, is a stark illustration of the measurement problem.

The measurement problem can be understood as the ostensible contradiction between two main rules of quantum theory. One rule states that systems undergo a continuous evolution in time described by a unitary transformation. The other states that, upon measurement, the state of the system changes abruptly according to the result of the measurement.

In the experiment, the friend owns at time t_1 a qubit prepared in the state $|+\rangle$ and at time t_2 measures it in the computational basis, obtaining either the result $|0\rangle$ or $|1\rangle$. Wigner, who (after hopefully asking for consent) had placed his friend and her whole lab in a sealed environment, gives a different account of the same events. Wigner assigns a Hilbert space $\mathcal{F}$ to the system comprising of his friend and her lab and assigns to it the pure state $|\text{ready}\rangle$ at time t_1. Even though it would be practically impossible to know the correct state, nothing stops Wigner from assuming that such a state exists. Since the lab is sealed, Wigner assumes that the lab and qubit system evolves unitarily according to:

$$\begin{aligned} |\text{ready}\rangle |0\rangle &\longmapsto |\text{zero}\rangle |0\rangle \, , \\ |\text{ready}\rangle |1\rangle &\longmapsto |\text{one}\rangle |1\rangle \, . \end{aligned} \tag{3.5}$$

Again, this evolution is impossible to compute in practice, but one can assume that such an evolution occurs, and whatever the state $|\text{zero}\rangle$ actually is, it is a state peaked around a semiclassical configuration in which the friend has seen the character '0' appear on her lab equipment. Since the qubit starts in the complementary state $|+\rangle$, Wigner deduces from the linearity of unitary evolution that the evolution of the lab is t_1 to t_2 leads to

$$|\text{ready}\rangle |+\rangle \longmapsto \frac{1}{\sqrt{2}} |\text{zero}\rangle |0\rangle + \frac{1}{\sqrt{2}} |\text{one}\rangle |1\rangle \, . \tag{3.6}$$

So, according to Wigner, not only the qubit is not in a definite state of the computational basis, but the whole lab is now in a superposition! How can two people give such contrasting accounts of the same events? Is there a way to decide if either one is right? Can they both be right, or is quantum mechanics incomplete? Finding a satisfactory solution to these questions is one of the tasks of the field of the interpretations of quantum mechanics, which we discuss next.

Note first however that if Wigner is right because quantum mechanics can be applied to all systems (including agents such as Wigner's friend), then it would in principle be possible to apply whatever unitary evolution lead to (3.6), and undo his friend's measurement. As we shall see in section 3.4.2, this has strong consequences on our notions of reality.

3.3 Different interpretations of QM

The search for an interpretation of quantum mechanics is an interdisciplinary effort between physics and philosophy. It raises complex issues in philosophy of science. For example, what are the requirements of a scientific theory, does it need to tell us something about reality, or should we just accept that the best we can do is predict the results of experiments? It also leads us to ask what it means for something to 'truly' exist. Additionally, the unavoidable presence of probability carries the question of the role of agency in formulating a scientific theory.

The search is completely ongoing, with many different contenders, often separated in vocal "camps," with little communication between camps [40, 185, 234, 245]. The situation is confounding because quantum theory is an extremely successful scientific theory. It sustains the technological foundations of our society, and it is used to formulate the most fundamental theory of physics, the standard model of particle physics, which is by all accounts an unparalleled success in the programme of reductionism.

Here we provide a brief (and partial!) overview of the field, by giving a short description of various positions, glossing over many nuances. As we will see, every solution comes at a price: each interpretation either leads to awesomely unfamiliar concepts, or relies on non-mainstream philosophical stances, and sometimes both.

3.3.1 Copenhagen, or no-interpretation, approaches

One way of approaching the measurement problem is to argue there is no problem to start with [107]. Quantum mechanics allows to predict with exquisite precision the results of experiments. It's a theory about 'detector clicks'. The quantum state is not a description of reality, but a summary of the information someone owns about a system and a tool to compute probability of measurement outcomes. In this view, the difference between Wigner's account and his friends' is just a consequence of an asymmetry of information. Wigner's friend knows more about the particle than Wigner himself, and this is why Wigner's state assignment differs from his friend's.

This approach is certainly the most used in practice and it is perfectly adequate for all practical purposes. However it amounts to a strong form of instrumentalism, for it abandons the hope that our best scientific theory can tell us about the world outside of our experimental interventions. Also, depending on how it is articulated, it can introduce a form of dualism, between "quantum systems" about which nothing can be said and the "macroscopic" world of people and detectors about which we compute probabilities.

A criticism often raised is that, supposedly, people and detectors are themselves made of atoms and thus should be amenable to a quantum mechanical description. Indeed, actual detectors are designed using quantum mechanical knowledge. When does a physical system stop being amenable to quantum mechanics and becomes a measurement apparatus? Similarly, one is lead to ask about Wigner's friend, and if she could ever be in a superposition. Following the instrumentalist spirit of this interpretation, one might answer that the problem is not well-posed: there is, in the end, a well-defined sense in which we inhabit a world obeying mostly classical laws. Wigner will never be able to do the experiments necessary to witness interference effects of superpositions of agents. The friend has seen a definite result, and Wigner is just ignorant of it. The next two interpretations provide opposite responses to this question.

3.3.2 QBism

Quantum mechanics, according to the QBist, is the best tool to guide *agents* in making decisions in our indeterministic world. Indeed, QBism [53, 108] holds that quantum mechanics is an extension of probability theory and, as such, should not be required to offer a picture of the world. This interpretation of QM is based on a specific interpretation of probability theory [76, 233], namely, that all probabilities are degrees of belief of a rational agents. Thus, the laws of quantum mechanics are not physical laws that govern the behaviour of physical systems; they are "laws of thought," guidelines on how to best structure one's beliefs based on previous beliefs and sensory data.

QBism is deeply rooted in American pragmatism and places decision-making agents at the centre of the world. Every single agent is welcome to start with their own assumptions and to use quantum theory to guide the updating of their beliefs in their decision-making process. QBists see the natural world as a stage where agents interact with and describe each other quantum mechanically. To every action taken by agent, the world responds with an unpredictable kick, which results in a new experience for the agent and something truly new in the world.

While the strong emphasis on subjective experience and agency makes QBism seem quite radical to many, the proponents raise an unobjectionable point: at the present moment, there is no sound and agreed-upon foundation for probability theory. The situation in the foundations of probability echoes that of quantum mechanics: the interpretation or probability used in practice, the frequentist interpretation, is not satisfactory. While it is true that we can use repeated experiments to learn about probabilities (and we do that all the time), we cannot *identify* probabilities and frequencies. This is because the frequency f of an outcome that has probability p only approaches p with *high probability* [179]. QBism takes this problem seriously: it says that we cannot understand quantum mechanics without understanding probability, and uses one of the proposed foundations of probability, subjective or personalist probability [233], as the foundation for interpreting quantum mechanics.

3.3.3 Relational quantum mechanics

Relational quantum mechanics (RQM) [221, 226] rejects instrumentalism. There is no special class of systems called "measuring apparata" that transform quantum possibilities in facts and conscious agents play no fundamental role. Quantum theory applies to everything. RQM holds that reality is built of discrete relational quantum events that can happen whenever any two systems interact.

It says reality has to be described relative to a given system, which serves as a *context* in which things happen (quantum variables assume definite values). The quantum state is seen as a computational tool used to compute the probability of quantum events in a certain context, given other quantum events in that same context. Any physical system can serve as a context in which variables take values.

However, the value of a variable is a *relative fact*, labelled by the system that serves as context.

RQM resolves the tension between the two rules of evolution (the continuous unitary evolution and the collapse at the moment of measurement) by saying that they are used to describe physics relative to different systems. When two systems A and B interact, their state relative to a third system C that does not take part in the interaction evolves continuously, while the state of A relative to B will update discontinuously whenever the interaction takes place: whenever the value of one of the variables of A takes a definite value relative to B.

In the Wigner's friend thought experiment, the value of the qubit at time t_2 is a fact for the friend, but not for Wigner. But according to RQM, there is no need for the friend to be a complex system, let alone sentient being, for the value of the qubit to actualise: as long as a variable of the friend is affected by the qubit, the qubit can take a value relative to the friend. And Wigner is not merely ignorant of this value: the qubit has no value relative to Wigner yet. Indeed, were Wigner to reason using classical logic as if he was just simply ignorant on the value of the qubit, he will be mislead in ignoring possible interference effects.

Part IV of this thesis is dedicated to RQM. In chapter 8, we will see that the failure of classical logic is understood as the result of ignoring the relational nature of reality and the reason relationality remained hidden until the XXth century relies on the uncontrolled interactions at our macroscopic scale, which "de-labels" facts and *stabilise* them. In chapter 9 we will explore in detail some counterintuitive consequences of the relativity of facts.

3.3.4 Everettian quantum mechanics

Everettian quantum mechanics (EQM) [254, 258] also holds that quantum mechanics applies to everything. However it disagrees with the previous interpretations on what the rules of quantum mechanics are. In particular EQM rejects the projection postulate and holds that all quantum evolution is unitary evolution, and proposes that one should understand the quantum state as a *representation* of the goings-on of the universe. This move has astonishing consequences: EQM says that the quantum state describes manifold, dynamically independent realities that keep multiplying and diversifying as a result of quantum mechanical uncertainty and entanglement.

Branching happens whenever a quantum uncertainty affects the macroscopic world. This happens not just as a result of laboratory experiments involving qubits but around every unstable atom, as a result of gamma rays colliding in the upper atmospheres of planets, and whenever classical chaotic dynamics amplifies quantum uncertainty to macroscopic scales. Decoherence makes it so that whenever a large enough system goes into a superposition of macroscopically different configurations, the branches are effectively decoupled, creating new worlds.[1] Note that most of the causes of branching does not create an integer number of branches, but a continuous amount of branches.

For example, the state of Wigner's friend, her lab and the quantum qubit at

[1]Hence the other famous name for this interpretation: the Many Worlds Interpretation.

time t_2,

$$\frac{1}{\sqrt{2}}\left|\text{zero}\right\rangle\left|0\right\rangle + \frac{1}{\sqrt{2}}\left|\text{one}\right\rangle\left|1\right\rangle, \tag{3.7}$$

says that there are now two versions of the friend and her lab, one in which she witnessed the qubit value 0 and one in which she saw 1. Because the dynamics of the lab is enormously complex, it is incredibly hard for these two *branches* to interact—to interfere, that is—and thus the two branches are effectively independent of each other. As soon as Wigner removes the seal from his friend's lab, the different interactions with the two branches of the friend's lab cause the nearby systems to branch, and this branching gradually propagates to the rest of universe.

The mind boggles when considering the worldview proposed by this interpretation, and this is continuously raised as a criticism of the theory. But it is crucial to understand that this unfathomable number of multiplying realities is not a *postulate* of EQM, but a *consequence* of its postulates. EQM only postulates that unitary quantum mechanics applies to all systems, and that the quantum state describes a state of affairs. As such, EQM is arguably quite frugal in its assumptions. It is also important to note that EQM has two big advantages over most other interpretations: it applies to all systems, and it naturally carries over to relativistic quantum field theory.

Finally, another often-heard criticism with EQM is that, since the evolution is deterministic and all alternatives happen in *some* branch, the theory cannot account for the probabilisitic aspect of quantum experiments. This criticism is based on initial uncertainty about this aspect of the theory, but it is now outdated, as the problem has been solved and now there exists a complete account for the rules of probability theory in relation to observed frequencies and decision theory [80, 120, 258]. Many of the problems in understanding probability in EQM are not problems specific to EQM, but problems with the philosophy of science highlighted by the strange context of EQM.

Interpreting the rules of quantum theory "on the nose" leads to unexpected and foreign pictures of reality. This has prompted some scholars to propose that quantum theory is incomplete and needs to be modified or supplemented with additional rules. We now see two examples.

3.3.5 The pilot-wave theory

Also known as the de Broglie-Bohm theory or Bohmian mechanics [91, 118], this interpretation posits that the quantum mechanical wavefunction is an incomplete description of a system.

A system system of N particles is described by a complex-valued wavefunction $\psi(\boldsymbol{q}_1, \cdots, \boldsymbol{q}_N) \equiv \psi(q)$ on the space of configurations, as well as the *actual* values $\hat{\boldsymbol{q}}_n$ of the positions of the particles. The wavefunction evolves unitarily according to the Schrödinger equation while it guides the actual positions of the particles according to:

$$\frac{d\hat{\boldsymbol{q}}_n}{dt} = \frac{\hbar}{m_n}\boldsymbol{\nabla}_n\phi|_{(\hat{\boldsymbol{q}}_1,\cdots,\hat{\boldsymbol{q}}_N)}, \tag{3.8}$$

where $\phi(q)$ is the phase of the wavefunction and $\boldsymbol{\nabla}_n$ is the gradient taken with respect to the coordinates of the nth particle. Notice that the velocity of a given particle might depend on the actual positions of all the other particles involved.

According to this interpretation, systems evolve deterministically for all times. Quantum uncertainty follows from the combination of the dynamics, classical ignorance of the actual positions, and the assumption of *quantum equilibrium*. The latter says that, in an ensemble of systems prepared in the state ψ, the actual positions of the particles are distributed in a (classically) random way described by the *quantum equilibrium measure*:

$$\rho(\hat{q})\mathrm{d}\hat{q} = |\psi(\hat{q})|^2 \mathrm{d}\hat{q}. \tag{3.9}$$

The quantum equilibrium distribution is preserved by the dynamics, namely the Schrödinger equation and the guiding equation (3.8), so that classical ignorance about a system is preserved over time. The result of a measurement depends on the actual position of the particle.

The pilot-wave model is appreciated as it describes a reality that is close enough to classical intuition, where particles have positions and they get pushed around by forces.[2] However it has its drawbacks. Note that, like in EQM, the wavefunction of the system never collapses and, as systems interact, the wavefunction becomes highly entangled. While decoherence makes interference terms disappear and suppresses the quantum effects from the guiding equation, the wave is still there. In particular, the function $\psi(q)$ will have support on vast regions of configuration space arbitrarily far from the actual configuration of the system. There is no agreement at the moment as to wether the wavefunction should be thought as part of reality, or merely as a useful way to state the laws that govern the movement of the particles (*i.e.* on whether it is ontic or nomological). Furthermore, the guiding equation for particle n depends instantaneously on the actual positions of all the other particles. The non-locality of dynamics is hidden by the randomness in quantum equilibrium, so that no signal can reliably be sent faster than light. However, the non-local dynamics is in tension with relativity theory as it picks out one preferred frame in which to formulate the dynamics, and adapting Bohmian mechanics to quantum field theory has not been successful yet.

One could in principle test the pilot-wave theory by finding systems out of quantum equilibrium, which will consistently violate the statistics computed with quantum theory (and possibly signal faster than light) [255].

3.3.6 Spontaneous collapse models

Another way to modify quantum theory is to posit an objective collapse mechanism so that the wavefunction of every particle has a small chance to collapse to a well defined state. There are several different models that differ on the basis of collapse (energy, momentum, basis), and the mechanism of collapse (irreducible or induced by interactions with an unknown field). But besides these details, the spontaneous collapse models all have similar features. A limpid and recent review is provided by Bassi *et. al.* in [15]. Here we provide a short overview of the Ghirardi-Rimini-Weber

[2]There is a way to formulate pilot-wave model in which particles obey Newton's second law, modified by the addition of a nonlocal *quantum potential*.

(GRW) model, which is the first model proposed, and perhaps the most well-known, as well as the Penrose-Diósi proposal for gravitationally-induced collapse.

GRW

In this model, the wavefunction $\psi_t(\boldsymbol{q}_1, \cdots, \boldsymbol{q}_N) \equiv \psi_t(q)$ of N particles evolves in time t according to the Schrödinger equation—most of the time. In any interval of time dt, each of the N particles has a probability $\lambda_{\mathrm{GRM}}\mathrm{d}t$ of spontaneously localising. When nth particle localises, the wavefunction instantaneously changes according to

$$\psi_t \longmapsto \frac{L_n(\boldsymbol{x})[\psi_t]}{\|L_n(\boldsymbol{x})[\psi_t]\|}, \tag{3.10}$$

where $L_n(\boldsymbol{x})$ is a linear operator whose action is defined by

$$L_n(\boldsymbol{x})[\psi](q) = \psi(q)e^{-\frac{1}{2}(\boldsymbol{x}-\boldsymbol{q}_n)/r_C^2}. \tag{3.11}$$

The operator $L_n(\boldsymbol{x})$ exponentially suppresses the parts of the wavefunction which have support on values of $\boldsymbol{q}_n$ far from the position $\boldsymbol{x}$. Thus, in the new wavefunction, the particle n is localised within an area a few r_{C} of diameter. The value of $\boldsymbol{x}$ itself is randomly chosen and its probability density is given by

$$p(\boldsymbol{x}|n,t) = \|L_n(\boldsymbol{x})[\psi_t]\|^2. \tag{3.12}$$

In this model, there are no point particles. The wavefunction is used to instead to compute the mass distribution associated with each particle. One can compute a *density of mass* for particle 1 at time t according to:

$$\rho_t^n(\boldsymbol{q}_1) = m_n \int \mathrm{d}^3q_2 \cdots \mathrm{d}^3q_N \, |\psi_t(q)|^2, \tag{3.13}$$

and similarly for the other particles. When a particle goes into an interferometer it really goes through both slits.

GRW has two free parameters, the rate of localisation λ_{GRW} and the localisation precision r_{C}. Standard values are

$$\lambda_{\mathrm{GRW}} = 10^{-16}\ \mathrm{s}^{-1}, \qquad r_{\mathrm{C}} = 10^{-7}\ \mathrm{m}. \tag{3.14}$$

The rate of localisation of each particle is extremely low but, since the localisation of each particle is independent, the rate of collapse grows linearly in the number of particles N and $N\lambda_{\mathrm{GRW}} \gg 1\ \mathrm{s}^{-1}$ for systems with a macroscopic number of particles. The Wigner friend thought experiment is resolved by saying that the state of the friend and the lab collapses extremely rapidly to one of the two branches. These values place the detection of spontaneous collapse just out of current technological capabilities, but the hope of the programme is to actually detect the consequences of this effect. If it becomes impossible to perform interferometric experiments with systems above a certain mass, even when accounting for all the standard sources of environmental decoherence, then this will be a clear signal that some form of spontaneous collapse is at play.

Penrose-Diósi gravitational collapse

The Penrose-Diósi model is a parameter-free version of spontaneous collapse. The model is of particular interest for us because it is motivated by the assumption that the gravitational field is a classical entity, incapable of being in a superposition. When the tension between the quantum superposition becomes "gravitationally" too large, the state collapses rapidly in a position-defined state.

A natural "gravitational" measure for the size of the superposition can be computed as follows. Suppose a system of mass m and mass density ρ is in a superposition of two states separated by $\boldsymbol{d}$. Then one might estimate the size of the superposition by

$$\Delta E_G = 8\pi G \left| \int d^3q \int d^3q' \; \rho(\boldsymbol{q}) \frac{\rho(\boldsymbol{q}') - \rho(\boldsymbol{q}' - \boldsymbol{d})}{|\boldsymbol{q} - \boldsymbol{q}'|} \right|, \tag{3.15}$$

the absolute value of the difference between the self-energy of a mass distribution $\rho(\boldsymbol{q})$ and the interaction energy between this mass distribution and one displaced by $\boldsymbol{d}$. Then the expected decay rate λ_{PD} for this superposition is:

$$\lambda_{\mathrm{PD}} = \frac{\Delta E_G}{\hbar}. \tag{3.16}$$

This parameter-free model has been recently falsified by underground experiments at Gran Sasso [**donadi2021underground**]. The spontaneous collapse results in stochastic motion, which in turn emits EM radiation, and results in heating. This is true for any state of matter, not just superpositions. The experiment in Gran Sasso attempted to measure this heat production of a germanium crystal, and the measured values ruled out the model with high confidence. There are other proposals for models of spontaneous collapse caused by a classical gravitational field [15]. All these models would be ruled out by the detection of gravitationally-mediated entanglement.

3.4 Experimental metaphysics

This proliferation of interpretations might seem to some an idle endeavour. Some theories are modifications of QM that have consequences that are, in principle, observable; but most of the interpretations are designed to give the exact same predictions as QM, so there seems to be no hope to distinguish them. Is this all useless metaphysics then? Not necessarily. The various interpretations offer different ways of thinking about the same set of observations. As Feynman was fond of pointing out, it is good to have different theoretical representations of the same physics, as we don't know which way of thinking will turn out to be most useful when we need to change the laws.

Surprisingly, there are *experimental* ways of constraining the various interpretations. By studying the logical structure of different kinds of theories, one can formulate general statements about which *classes* of theories are compatible with observations. These often *quantitative* results are the main output of the field of experimental metaphysics [48, 50]. The most celebrated are of course Bell's two theorems [16, 19]. Other notorious results include the Kochen-Specker theorem

[148], Cirel'son's bound [64], the PBR theorem [213], and the Local Friendliness inequalities [32]. The development of such no-go theorems is one of the tools of quantum foundations research and they lead to clarifications of certain obscure aspects of the quantum, as well as practical applications.

Here, we will review Bell's second theorem [19, 20] that shows that quantum physics contains a form of irreducible non-locality and characterises what is special about entanglement. We will also review the recent no-go theorem about the absoluteness of events in quantum physics by Bong *et. al.* [32].

3.4.1 Bell

Bell's 1976 theorem puts quantum theory in strong tension with the intuitive notion named *local causality*, namely, the idea that correlations between two spacelike separated events are to be explained in terms of events that happened in their common past. This condition implies quite general constraints on the strength of statistical correlations of random variables that share a common cause. These constraints are formulated in terms of inequalities, known as Bell inequalities. Not only does quantum theory predict the violations of these inequalities, the inequalities are routinely violated in quantum labs all over the world.

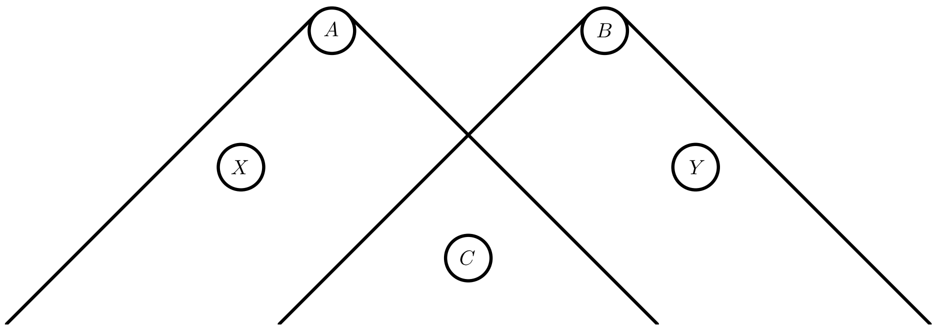


Figure 3.1. Spacetime view of the Bell setup.

In the Bell setup, one considers the correlations of variables A and B observed at two spacelike separated spacetime regions and how these correlations depend on the values of other variables X, Y and C, where C is in the causal past of both A and B, X is in the causal past of A but not B, and the opposite is true for Y. The situation is displayed in figure 3.1. X and Y are assumed to be *freely chosen*, meaning that they are not correlated with anything outside their future light-cone. Let the numbers

$$f(abxy) \tag{3.17}$$

describe the observed frequencies of the various outcomes, where we denote by a a specific value of A and similarly for the other variables. We are interested in the correlations between A and B for different values of X and Y, that is, in the distribution

$$f(ab|xy). \tag{3.18}$$

Now, by the principle of local causality, there should be some C in the past of both A and B that explains the correlations, namely, such that

$$f(ab|xy) = \sum_c f(ab|cxy) f(c|xy) = \sum_c f(a|xyc) f(b|xyc) f(c|xy). \tag{3.19}$$

Note that the first equality is trivial, while the second equality is the expression of the principle of local causality. Since X and Y are freely chosen, we also have the simplifications

$$f(a|xyc) = f(a|xc), \tag{3.20}$$
$$f(b|xyc) = f(b|yc), \tag{3.21}$$
$$f(c|xy) = f(c), \tag{3.22}$$

so that, finally

$$f(ab|xy) = \sum_c f(a|xc) f(b|yc) f(c). \tag{3.23}$$

The formula above has been derived assuming only the principles of local causality and free choice and applying them to the situation in figure 3.1. It relies on no particular assumption about the physical interpretation of the variables A, B, C, X, and Y other than they take values in the respective spacetime region.

Equation(3.23) imposes constraints on the correlations between A and B. For example, if we assume that A and B can only take the values[3] -1 and 1, then the numbers

$$\mathrm{C}(x, y) = \sum_{ab} ab \cdot f(ab|xy), \tag{3.24}$$

which quantify the correlations between A and B for different values of X and Y, can be shown to obey the following inequality,

$$S \equiv \left|\mathrm{C}(x_0, y_0) + \mathrm{C}(x_0, y_1) + \mathrm{C}(x_1, y_0) - \mathrm{C}(x_1, y_1)\right| \leq 2, \tag{3.25}$$

where x_0 and x_1 are any two values of X, and y_0 and y_2 any two values of Y. This is known as the Clauser, Horne, Shimony, and Holt (CHSH) inequality [65].

Now, it is well-known that there are quantum systems that violate the above inequality. Spin measurements on the singlet state

$$|\psi\rangle = \frac{1}{\sqrt{2}} |\uparrow\downarrow\rangle - \frac{1}{\sqrt{2}} |\downarrow\uparrow\rangle, \tag{3.26}$$

where X and Y select the axis of spin measurement of each particle, and $A, B = \pm 1$ depending if the result is 'up' or 'down' can achieve, for appropriate choices of measurement axes allow a maximal value of

$$S = 2\sqrt{2}. \tag{3.27}$$

Thus quantum theory—as well as observational data [7, 117, 128]—is incompatible with the notions of local causality and free-choice. Every interpretation has to make sense of this fact.

[3]Note that this can always be done by coarse-graining the observations in two different regions.

One way to tackle this interpretational problem, as argued in the excellent review [264], is to break down local causality in two ideas. First, the idea of *relativistic causality*, namely, that the causes of an event are to be found in its past lightcone. Second, *Reichenbach's principle* of decorrelating explanation, which states that a correlation between A and B is *explained* by a variable C if conditioning on C removes the correlation. Relativistic causality together with Reichenbach's principle imply local causality, thus rejecting one or the other is a way to reconcile one's picture of the world with observations. Both options are quite radical. Abandoning relativistic causality so that (3.20) fails, is in stark conflict with the spirit of special relativity. This is the way taken by pilot wave theory, spontaneous collapse models, and some readings of the no-interpretation interpretation, for example. There are general results about the fine-tuning required so that models that change the causal structure of these Bell-type experiment do not lead to superluminal signalling [49, 266]. Other approaches, including other flavours of the no-interpretation position, take the violation of the Bell inequalities as a failure of Reichenbach's principle. However, it is not quite clear what should replace it [51, 158].

One might object to some more basic premise of Bell's proof. For example, one might say that the simplification (3.22) is unwarranted, as there might be a common cause to the outcomes of X, Y, and C, so that they are not actually independent. This idea is often called *superdeterminism* and has serious proponents [1, 138]. However, it is an empirical fact that one can set up three random number generators that display no correlations. The existence of their common cause would only show up when these random variables are used in Bell-type experiments. This approach indeed faces the same fine-tuning problem as rejecting relativistic causality [49, 266].

Another way out is to reject the premise that it makes sense to speak of variables taking values in given spacetime regions, without conditioning this statement on anything else. This is called rejecting *macroreality*, or the *absoluteness of observed events*. QBism, relational quantum mechanics (RQM), and Everettian quantum theory all do this, in different ways. Indeed, QBism and RQM both say that it does not make sense to describe the experiment in such absolute terms. No agent (for QBism) or single system (RQM) can experience two spacelike separated events, instead, they would have to wait for the result of both experiments to arrive in the same spacetime region, at which point the correlations are not in conflict with local causality [108, 169]. However, in both interpretations there is a sense in which all observers inhabit an emergent macroreality, and this macroreality is still in violation of local causality, so these interpretations might still need to reject Reichenbach's principle, suitably reformulated. The issue is still not completely resolved [50, 204]. In contrast, EQM is compatible with variables taking values at specific spacetime locations, but they do not take *single* values. The apparata measuring the spins will go into a superposition, and there will be *local* branching. When the signal from one apparatus reaches (in a superposition) the other apparatus, there is branching again. This allows for a completely local description of events; see [35, 81, 258].

3.4.2 Bong *et. al.*

Rejecting the existence of a fundamental macroreality may seem an extravagant way out of Bell's theorem. However, it is a more natural option to respond to Bong *et. al.*'s recent no-go theorem [32], which pitches macroreality, no-superdeterminism, locality and quantum mechanics against each other.

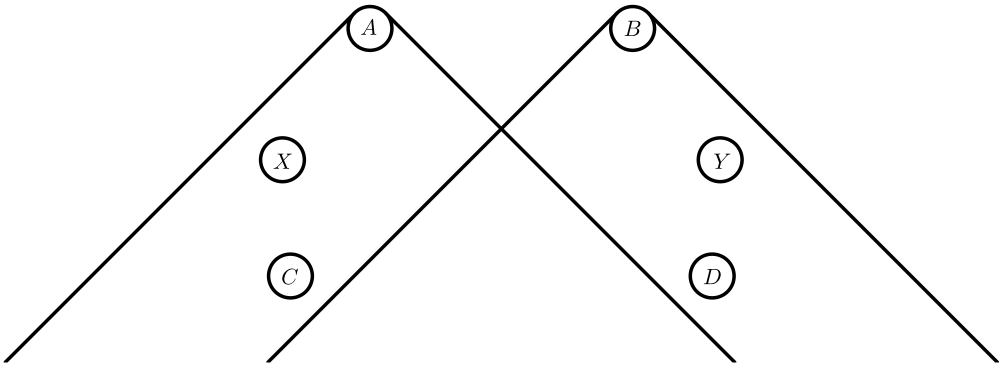


Figure 3.2. Spacetime view of the extended Wigner's friend setup.

The setup is an extension of the Wigner's friend thought experiment, see figure 3.2. In this case we have two "Wigners"—Alice and Bob—who each has their own friend in a sealed lab—Charlie and Debbie, respectively. The procedure is as follows. Charlie and Debbie each measure a quantum system in their lab, the results are the random variables C and D. We assume that the systems are always prepared in the same way, and the measurements are performed on the same basis for all trials. Then, Alice and Bob sample the random variables X and Y. If $X = 1$, Alice simply asks Charlie what the result of the experiment was and sets A to whatever Charlies says, otherwise she performs another kind of experiment and selects A this way. Similarly for Bob, Debbie, Y and B.

We are again interested on putting bounds on the correlations of A, B, X, and Y, that is, on the frequencies

$$f(ab|xy). \tag{3.28}$$

However, if we also assume that C and D take definite values, we postulate that there exists a distribution

$$\tilde{f}(abcd|xy) \tag{3.29}$$

that accounts for f in the sense that

$$f(ab|xy) = \sum_{c,d} f(abcd|xy). \tag{3.30}$$

Additionally, since Alice sets A to whatever value Charlie says when $X = 1$ (and similarly for Bob), we require also

$$\tilde{f}(a|cd, x = 1, y) = \delta_{a,c} \quad \text{and} \quad \tilde{f}(b|cd, x, y = 1) = \delta_{b,d}. \tag{3.31}$$

These last three equations follow from the assumption of *macroreality*, or *absoluteness of observed events*: it makes sense to think of the values of C and D, observed by

Charlie and Debbie, as existing regardless of what Alice and Bob choose to do. We then assume *no superdeterminism*, meaning that it is possible to pick X and Y so as to be independent of C and D,

$$\tilde{f}(cd|xy) = f(cd). \tag{3.32}$$

We also assume *locality*, so that A cannot depend on Y and B cannot depend on X, even when accounting for the unseen values of C and D:

$$\tilde{f}(a|cdxy) = \tilde{f}(a|cdx) \tag{3.33}$$
$$\tilde{f}(b|cdxy) = \tilde{f}(b|cdy) \tag{3.34}$$

These equations for $\tilde{f}$ imply constraints on f and on the correlations between A and B. These constraints are known as the *Local Friendliness* (LF) inequalities. Local friendliness is the conjunction of macroreality, no-superdeterminism, and locality.

Bong *et. al.* showed that, if one assumes that quantum theory can be applied to all systems, including experimenters such as Charlie and Debbie, then the LF inequalities can be violated. The quantum experiments that violate the LF inequalities require Charlie and Debbie to each own one part of a bipartite entangled system. They also require that Alice and Bob are capable of rewinding the evolution in their friend's lab when X or $Y \neq 1$. In that case they then perform measurements directly on their friend's entangled system, measurements that are complementary to those that their friends performed.

Contrary to Bell's inequality, the violation of the LF inequality has not been established to the satisfaction of the community yet. Experiments violating LF inequalities have been performed [32, 212] with the spatial mode of a photon acting as Charlie or Debbie. Since we are used to having photons in superpositions of spatial modes, most physicists doubt that these are instances of *bona fide* violations of the LF inequalities. On the other hand, performing the experiment with a human might remain forever out of reach. However, with advances in quantum control of matter, the experiments will be performed with more and more complex systems acting as "friends." A day might come when intelligent computer programmes acting as agents running on quantum computers will be put in such superpositions.

Perhaps quantum mechanics fails at some scale and, once a system reaches a certain level of complexity it does not obey quantum physics any longer. If quantum mechanics *is* universal however, the theorem implies that the world does not allow for local friendliness. Given the empirical success of quantum theory, one ought to take the consequences of the theorem seriously.

Interpretations that already accepted superdeterminism or let go of relativistic causality, escape the Bong *et. al.* theorem the same way (relativistic causality implies locality). However, now it's not enough to reject Reichenbach's principle. So the no-interpretation approaches have to find another solution, either rejecting one of the three assumptions, or admit that quantum mechanics cannot be applied to observers. QBism, RQM and EQM all reject the existence of observer-independent facts.

Part II

Low Energy Quantum Gravity

Chapter 4

Quantum optics simulation of a quantum gravity experiment

Quantum gravity predicts that two masses in a superposition of position will become entangled as a consequence of gravitational interaction. This effect could be created in “low energy” experiments that may be realised in the near term. Detection of gravitationally mediated entanglement (GME) would be the first direct observation of a quantum gravity prediction. What’s more, GME cannot happen if gravity is mediated by a classical system [112] (see section 2.5). Given that to date we have zero experimental evidence for quantum gravitational phenomena, this experiment is thus of immense interest for basic research in physics. It showcases the usefulness of quantum information techniques in quantum gravity explorations and opens a novel avenue for quantum gravity phenomenology.

As a preparation to the anticipated actual experimental realisation, it is crucial to clarify the implications of possible experimental outcomes and discuss experimental techniques. Quantum simulators [99, 114, 123], quantum systems that can be manipulated to mimic the dynamics of harder to control quantum systems, provide powerful tools for this purpose. The advantage of a quantum simulation over a classical computer simulation is that it allows to verify and highlight the fundamental physical principles that underlies the effect under investigation.

In this chapter, we present two quantum logic simulators of the GME experiment and their implementation using photonic degrees of freedom. The experimental implementation of the simulators is still undergoing, as there were important delays caused by the pandemic, but we report results from one of the simulators [210]. The simulators can be used as a test to show how to certify the non-classicality of the field given realistic levels of noise, by using an entanglement witness and/or the violation of Bell’s inequalities. One of the simulators is also able to replicate the effects of collapse models by introducing decoherence to the system. In this case, state tomography can be employed to verify the absence of entanglement.

Quantum simulation can be performed through different physical systems, such as superconductive systems [139], nuclear magnetic resonance (NMR) quantum processors [90], trapped ions [29] and photons [9]. A preliminary simulation of GME appeared in [25], which uses nuclear magnetic resonance. The simulation we propose is closer to the actual physics of the GME experiment.

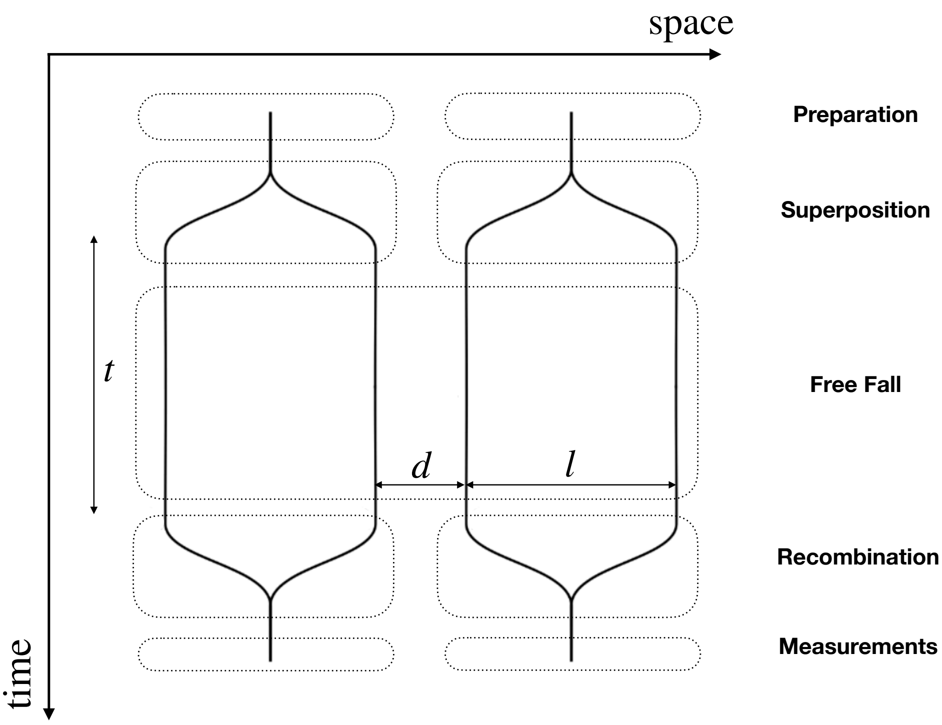


Figure 4.1. Sketch of the simulated experimental setup as proposed in [34]. Two masses are set in path-dependent superposition and get entangled as a result of gravitational interaction. See section 1.4.1 for a longer description.

4.1 The GME experiment

Let us briefly recall the implementation proposed in [34], and discussed in more detail section 1.4.1. The experiment features two adjacent interferometers, each travelled by a mass, and consists of five stages; see Figure 4.1. During the Preparation stage, two nanodiamonds of mass m with embedded magnetic spin-$\frac{1}{2}$ oriented along the x-axis, are released from a magnetic trap. In the Superposition stage a series of EM pulses modifies the position of the masses depending on z-component of the embedded spin, resulting in each mass being in a spin-dependent path superposition, at rest at a distance $l/2$ from their initial position. During the Free Fall stage, the masses are in free fall for a time t. In the Recombination stage, another series of EM pulses undoes the path superposition. The particles are collected during the Measurement stage and a spin measurement is applied. Repeated runs of the experiment allow to certify the presence of entanglement by studying the spin correlations. The setup is such that electromagnetic interactions between the masses are negligible compared to the gravitational interaction, so that if entanglement is present, it was mediated by gravity. We now proceed to identify the relevant degrees of freedom and their physical meaning.

The GME can be understood as a consequence of a macroscopic superposition of geometries, as already argued by [62, 63], see also section 1.4.1. Let us write $|g_{XY}\rangle = |XY\rangle\,|h_{XY}\rangle$, where X and Y denote the positions of the masses (C, L, or

R) and $|h_{XY}\rangle$ denotes the state of the metric perturbation field peaked around the corresponding classical solution. Then $|g_{XY}\rangle$ is the state of the spacetime degrees of freedom of the system. The evolution of the state of the system after each stage is:

$$|\psi\rangle_{\mathrm{P}} = \left(|\uparrow\rangle + |\downarrow\rangle \right) \otimes \left(|\uparrow\rangle + |\downarrow\rangle \right) \otimes |g_{CC}\rangle \tag{4.1}$$

$$|\psi\rangle_{\mathrm{S}} = |\uparrow\uparrow\rangle\, |g_{LR}\rangle + |\uparrow\downarrow\rangle\, |g_{LL}\rangle + |\downarrow\uparrow\rangle\, |g_{RR}\rangle + |\downarrow\downarrow\rangle\, |g_{RL}\rangle \tag{4.2}$$

$$|\psi\rangle_{\mathrm{FF}} = e^{i\phi_{LR}}\, |\uparrow\uparrow\rangle\, |g_{LR}\rangle + e^{i\phi_{LL}}\, |\uparrow\downarrow\rangle\, |g_{LL}\rangle + e^{i\phi_{RR}}\, |\downarrow\uparrow\rangle\, |g_{RR}\rangle + e^{i\phi_{RL}}\, |\downarrow\downarrow\rangle\, |g_{RL}\rangle \tag{4.3}$$

$$|\psi\rangle_{\mathrm{R}} = \left(e^{i\phi_{LR}}\, |\uparrow\uparrow\rangle + e^{i\phi_{LL}}\, |\uparrow\downarrow\rangle + e^{i\phi_{RR}}\, |\downarrow\uparrow\rangle + e^{i\phi_{RL}}\, |\downarrow\downarrow\rangle \right) \otimes |g_{CC}\rangle\,. \tag{4.4}$$

During the Free Fall stage, the geometry of each quantum branch is well approximated by a static configuration of two masses held at a fixed distance [54]. In each branch, the geometry is in a coherent state peaked on a macroscopic (classical) geometry. The four different positions of the masses in the superposition correspond to four diffeomorphically inequivalent static states of the geometry. Thus, the position and spins of the masses are entangled with the spacetime geometry.

This analysis highlights that the creation of entanglement between the spins is a consequence of the geometry itself being in a superposition during the experiment. It also makes clear that we only need a low-dimensional Hilbert space to model the coherent states of the gravitational field that are relevant during the experiment. In fact, we can simulate the gravitational field with a two qubit space. The spin degrees of freedom will require another two qubits.

For the rest of the work, we take the approximation $l \gg d$, so we take into account only the phase generated on the branch of closest approach, $|RL\rangle$. This is not likely to be the case in the actual experiment, but it simplifies the simulation without harming the logic of the experiment. We thus take

$$\phi_{LL} = \phi_{RR} = \phi_{LR} = 0, \tag{4.5}$$

and write $\phi = \phi_{RL}$.

Thus, the state of the spins at the moment of measurement will be

$$|\psi\rangle_{\mathrm{M}} = |\uparrow\uparrow\rangle + |\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle + e^{i\phi}\, |\downarrow\downarrow\rangle\,. \tag{4.6}$$

4.2 Quantum simulators

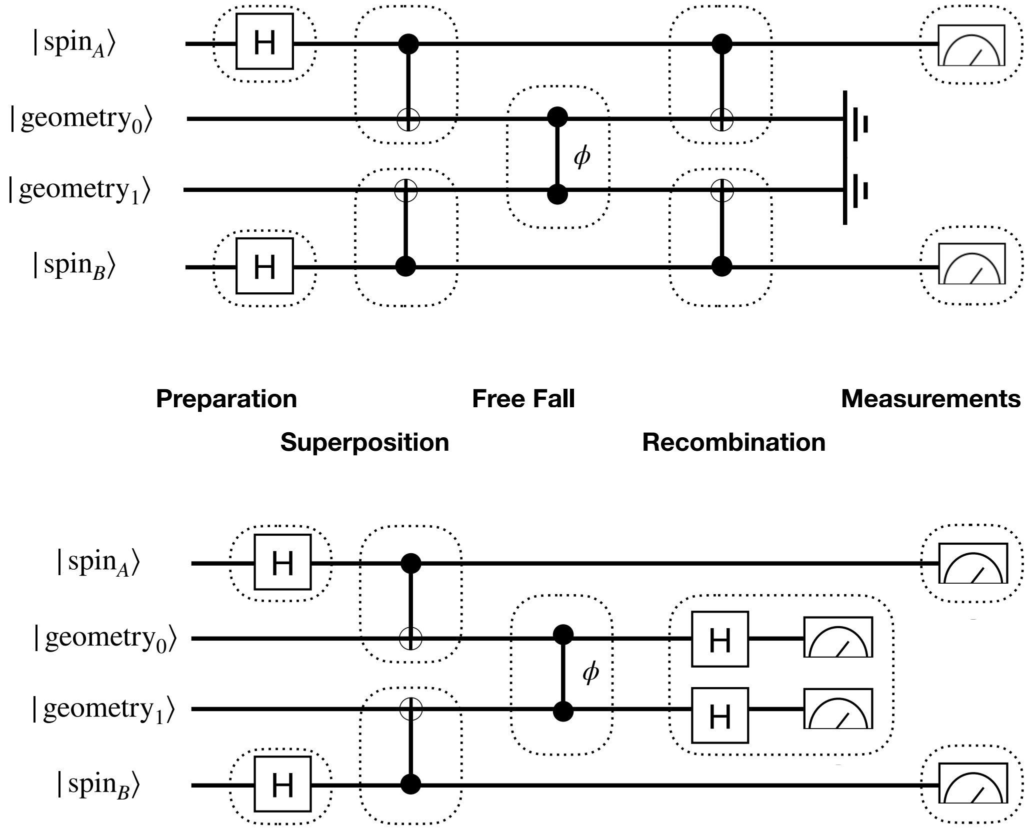


Figure 4.2. Quantum circuits for simulators. The two proposed simulators are presented here in circuit diagram form. In both cases, two qubits represent the spin degrees of freedom, while two qubits represent the geometry. The Quantum Circuit (QC) Simulator. top, is meant to simulate low energy quantum gravity. In the Post Selection Quantum Circuit (PSQC) Simulator, bottom, operating on the geometry qubits and post-selecting allows to additionally simulate the effects of decoherence.

We now map the experiment on a quantum circuit. We model the system of the two spins and the geometry as a 16-dimensional system. Each embedded spin is simulated with a qubit, while the geometry degrees of freedom with two qubits, a ququart. We write vectors as belonging to the following total Hilbert space:

$$\mathbb{C}^2 \otimes \mathbb{C}^4 \otimes \mathbb{C}^2 = \mathcal{H}_{\mathrm{spin}_A} \otimes \mathcal{H}_{\mathrm{geometry}} \otimes \mathcal{H}_{\mathrm{spin}_B}. \tag{4.7}$$

What allows us to model the geometry degrees of freedom with a finite dimensional Hilbert space is that in the five stages of the experiment, only a few states of the geometry come into play so that a digital quantum simulation is able to capture the essence of the process.

4.2.1 Quantum Circuit simulator

The Quantum Circuit (QC) Simulator of (see top of figure 4.2) is a straightforward representation of the dynamics of the GME experiment.[1] The guiding principle is to recognise that the state of the geometry after the superposition stage (1.18) depends entirely on the state of the spins: the state of the spin is "written" in the geometry. Two qubits represent the spin degrees of freedom of the nanodiamond. A ququart (two qubits) are assigned to the geometry as four coherent states of the geometry are relevant.

In the Preparation stage, Hadamard H gates are applied to the spin qubits to set them in the state $|+\rangle = |0\rangle + |1\rangle$, corresponding to the spin in the positive x-direction. The full state of the system at the end of the Preparation stage is

$$|+\rangle\,|00\rangle\,|+\rangle\,. \tag{4.8}$$

In the Superposition stage the two CNOT gates entangle the ququart with the spins, sending the above state into:

$$|0000\rangle + |0011\rangle + |1100\rangle + |1111\rangle\,. \tag{4.9}$$

The Control Phase gate mimics the Free Fall stage, adding a relative phase only to the branch where the gravitational ququart is in the state $|11\rangle$:

$$|0000\rangle + |0011\rangle + |1100\rangle + e^{i\phi}\,|1111\rangle\,. \tag{4.10}$$

Finally, in the Recombination stage, the CNOT gates are applied again, disentangling the geometry ququart from the spin qubits:

$$|0000\rangle + |0001\rangle + |1000\rangle + e^{i\phi}\,|1001\rangle\,, \tag{4.11}$$

so that the state of the spin qubits is

$$|00\rangle + |01\rangle + |10\rangle + e^{i\phi}\,|11\rangle\,. \tag{4.12}$$

There is a faithful mapping between (4.1) to (4.6) and (4.8) to (4.12). The second and third qubits represent the geometry ququart, and the remaining two the spin qubits. For instance in Eq.(4.10) we have: $|0000\rangle \leftrightarrow |\uparrow\uparrow\rangle\,|g_{LR}\rangle$, $|0011\rangle \leftrightarrow |\uparrow\downarrow\rangle\,|g_{LL}\rangle$, $|1100\rangle \leftrightarrow |\downarrow\uparrow\rangle\,|g_{RR}\rangle$ and $|1111\rangle \leftrightarrow |\downarrow\downarrow\rangle\,|g_{RL}\rangle$.

4.2.2 Post Selection Quantum Circuit simulator

In the second simulator (figure 4.2 bottom), we apply single-qubit gates on the geometry ququart after the free fall phase and measure them, keeping the results only if a certain outcome is achieved. If the geometry ququart is projected on the $|{+}{+}\rangle$ state, then the state of the spin qubits is (4.12), thus obtaining the final result of the previous simulator. Additionally, the photonic implementation of this simulator will allow us to simulate the effects of decoherence.

[1] This quantum circuit also appeared in [25], although it was not implemented.

4.2.3 Simulating gravitationally induced decoherence

As we mentioned in section 3.3, there is a breadth of spontaneous collapse models [15]. While these models differ in their motivations, mechanisms and in some quantitative details, they produce the same qualitative effect: a multi-particle system in a centre of mass superposition does not retain coherence for macroscopic timescales; the superposition becomes a classical mixture. One of the motivations of these models is to prevent macroscopic superpositions (thus solving the measurement problem beyond explaining why interference effects become negligible, which is already explained by decoherence). Even though most of these models contain at least one free parameter that determines the rate of collapse, they all expect a particle this size to collapse rapidly and thus GME should not take place. Detecting GME will render these models implausible.

Phenomenologically, the spontaneous collapse acts as a dephasing channel in the position eigenbasis. That is, if a particle starts off in a superposition described by the state $|\psi\rangle = (|x_1\rangle + |x_2\rangle)/\sqrt{2}$, after a certain time, its state undergoes a transformation:

$$\rho = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \longmapsto \frac{1}{2}\begin{pmatrix} 1 & e^{-\gamma} \\ e^{-\gamma} & 1 \end{pmatrix}, \tag{4.13}$$

where γ will be some model-dependent positive number that depends on the composition and geometry of the system, the size of the superposition, and the elapsed time.

In the experiment, each of the two masses would be independently undergoing spontaneous collapse. The spins start in a pure state, but since they become entangled with the position of the masses, when the system collapses on the position basis, the spins also collapse. Thus, the spins would not be in the state of equation (4.12),

$$\frac{1}{4}\begin{pmatrix} 1 & 1 & 1 & e^{-i\phi} \\ 1 & 1 & 1 & e^{-i\phi} \\ 1 & 1 & 1 & e^{-i\phi} \\ e^{i\phi} & e^{i\phi} & e^{i\phi} & 1 \end{pmatrix} \tag{4.14}$$

Instead, they will be in the state partially mixed state

$$\frac{1}{4}\begin{pmatrix} 1 & e^{-\gamma} & e^{-\gamma} & e^{-2\gamma-i\phi} \\ e^{-\gamma} & 1 & e^{-2\gamma} & e^{-\gamma-i\phi} \\ e^{-\gamma} & e^{-2\gamma} & 1 & e^{-\gamma-i\phi} \\ e^{-2\gamma+i\phi} & e^{-\gamma+i\phi} & e^{-\gamma+i\phi} & 1 \end{pmatrix}. \tag{4.15}$$

Thus, provided all known sources of decoherence are taken into account, the experiment can also in principle be used to detect spontaneous collapse.

In the PSQC simulator, if we do not postselect on the basis of the measurement on the geometry ququart, the state of the spin qubits at the moment of measurement will be

$$\rho_{\text{mix}} = \frac{1}{4}\left(|00\rangle\langle 00| + |01\rangle\langle 01| + |10\rangle\langle 10| + |11\rangle\langle 11| \right) \tag{4.16}$$

This simulates the effect of complete decoherence. To simulate only partial decoherence and obtain the state (4.15), we apply dephasing channels on the geometry qubits before the post-selection. The action of a single-qubit dephasing channel is:

$$D_\gamma : |i\rangle\langle j| \longmapsto e^{-\gamma(1-\delta_{ij})}\, |i\rangle\langle j| \tag{4.17}$$

and thus

$$D_\gamma \otimes D_\gamma : |im\rangle\langle jn| \longmapsto e^{-\gamma(1-\delta_{ij}-\delta_{mn})}\, |im\rangle\langle jn|\,. \tag{4.18}$$

4.2.4 Entanglement certification

Measuring an entanglement witness and violating a Bell inequality are two ways to certify the presence of entanglement. The entanglement witness requires fewer measurements and can certify entanglement in states that are not capable of violating the Bell inequalities. Violations of the Bell inequalities, on the other hand, is a device independent way of certifying the presence of entanglement, meaning that if a violation is measured, one can conclude the presence of entanglement with minimal assumptions about the functioning of the apparatus.

In the following, we measure the witness proposed in the original paper [34], which, in our setup, is

$$\mathcal{W}_B = 1 - \langle \sigma_x \otimes \sigma_z \rangle - \langle \sigma_y \otimes \sigma_y \rangle. \tag{4.19}$$

For the final state of the simulators (4.12), it yields $\mathcal{W}_B = \cos\phi$. We also violate the CHSH inequality by measuring σ_x and σ_z on one spin qubit and $(\sigma_x + \sigma_z)/2$ and $(\sigma_x - \sigma_z)/2$ on the other.

Demonstrating that no entanglement is present is harder. We intend to do this by performing quantum state tomography [144], thus reconstructing the quantum state.

4.3 Photonic implementation of the simulators

Having specified the abstract structure of the two simulators, we now describe their proposed implementation.

4.3.1 Photonic implementation of the Quantum Circuit simulator

The implementation of the QC simulator is shown in Fig. 4.3. Two photons carry the four logical qubits. The polarisation of each photon carries a spin qubit of the simulator, while the path degree of freedom carries the geometry ququart. The initial and final entangling gates, acting on path and polarisation of single photons, are realised by polarising beam splitters. The Control Phase gate acting on the paths of the photons is realised by a probabilistic scheme exploiting bosonic interference [186]. This will only induce a $\phi = \pi$ phase. We are also investigating the possibility to implement the tunable control phase gate reported in [249] in order to simulate various amounts of free-fall times [75].

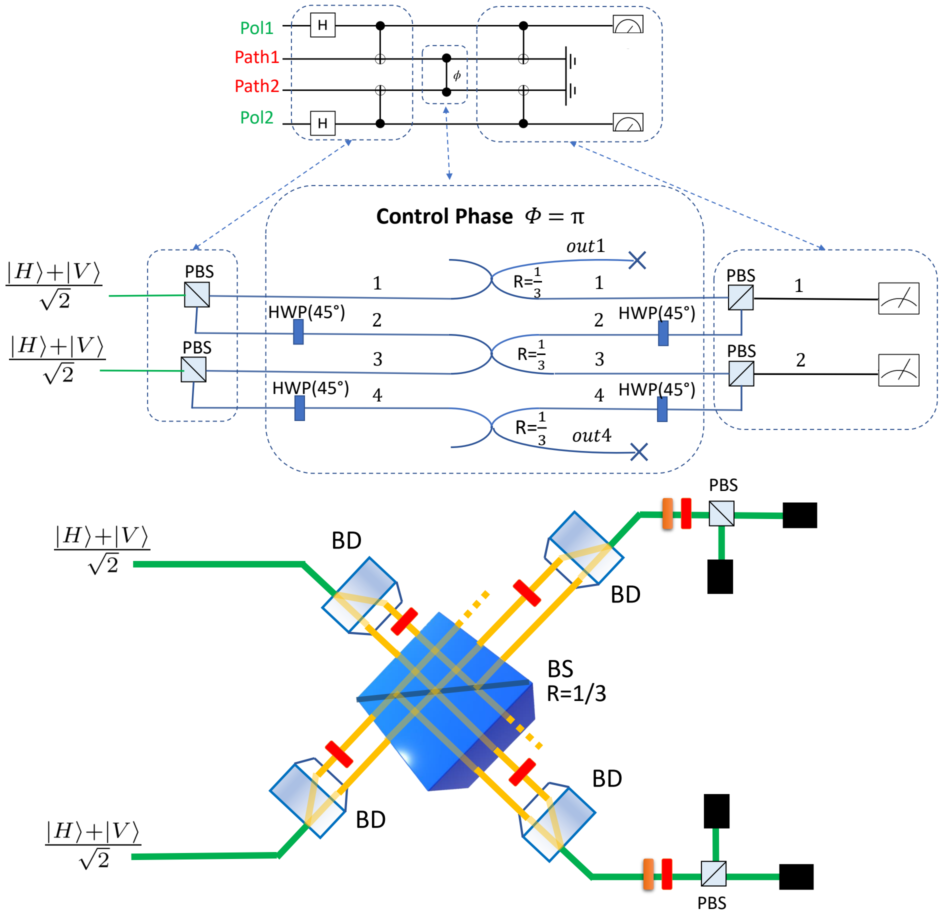


Figure 4.3. Two Photon Implementation of the Quantum Circuit Simulator (QC). The spin qubits of the simulator are encoded in the polarisation degrees of freedom of the two photons, while the geometry degrees of freedom are encoded in their paths. The two photons are independently prepared in a superposition of horizontal and vertical polarisation and go through a polarising beam splitter, which completely entangles the path of each photon with its polarisation. The Control Phase gate is implemented thanks to bosonic interference due to the indistinguishability of the photons. Two half-waveplates momentarily make the polarisation of all paths equal in order to allow the realisation of the control-Phase gate on this degree of freedom. Finally, the qubit state is restored by two other half-waveplates and the paths are recombined by final polarising beam splitters that disentangles path and polarisation of the two photons.

4.3.2 Photonic implementation of Post Selection Quantum Circuit simulator

The implementation of the PSQC simulator is shown in Fig. 4.4. In this case, the four logical qubits are carried by the polarisation degree of freedom of four different photons. Two maximally entangled pairs of photons are generated by two independent SPDC sources. This implements the Preparation and Superposition stages. The Free Fall stage is implemented via a probabilistic CZ gate employing a polarisation-dependent beamsplitter [147, 153, 189]. This gate acts on one photon from each pair. Postselecting on the result of measurements of these two photons allows the simulation of the Recombination stage.

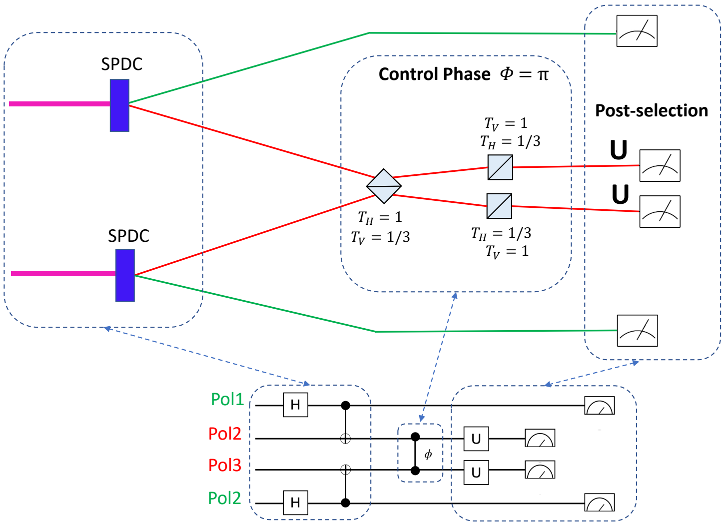


Figure 4.4. Four Photon implementation of the Post Selection Quantum Circuit Simulator. Each qubit of the simulator is encoded in the polarisation degree of freedom of a photon. The Preparation and Superposition stages are directly implemented by the creation of two maximally entangled photon pairs by SPDC process in non-linear crystals. The Control Phase gate on the two geometry qubits (polarisation degree of freedom of two photons) is implemented thanks to partially polarising beams splitters with transmittivities T_H and T_V relative to horizontal and vertical polarisations, respectively. The polarisation of the photons carrying the geometry qubits (red lines) is measured on bases defined by an operator U and the results are used for postselection.

4.3.3 Decoherence simulation

In both simulations described above, we can introduce decoherence in the states of the geometry by coupling different branches of the states to different delays, using birefringent materials in which photons of different polarisation travel at different speeds. When the delay information is ignored, this results in decoherence of the state. The delay effectively acts as an environment degree of freedom.

Consider the PSQC Simulator. The initial state of the polarisation is the product of two maximally entangled states. Focusing on one pair, the state is $|\Psi^-\rangle = (|H\rangle\,|V\rangle - |V\rangle\,|H\rangle)/\sqrt{2}$, corresponding to the state of one spin and part of the geometry. If the photon corresponding to the geometry qubit passes through a birefringent slice, the resulting state is

$$|\Psi\rangle_{\mathrm{del}} = \frac{1}{\sqrt{2}}(|H\rangle\,|V\rangle_{t_V} - |V\rangle\,|H\rangle_{t_H}), \tag{4.20}$$

where t_H and t_V are the distinguishable delays acquired by the horizontal and vertical polarisations, respectively. If the delay is greater than the coherence time of the photons (that is, if the delay is larger than the width of the photon packets), tracing out the information about delays results in a completely depolarised state:

$$\rho_{\text{mix}}^{\text{pol}} = \frac{1}{2}\big(\,|HV\rangle\!\langle HV| + |VH\rangle\!\langle VH|\big). \tag{4.21}$$

If instead the delay is less than the coherence time of the photons, the state will be a partially entangled one:

$$\rho_f = \nu\,|\Psi^-\rangle\!\langle\Psi^-| + (1-\nu)\rho_{\text{mix}}^{\text{pol}}, \tag{4.22}$$

where ν is a parameter quantifying the residual coherence after the delay between polarisations. Changing the thickness of birefringent slices allows to vary ν, going from a pure state to a completely decoherent case, simulating possible decoherence effects on spatial superpositions of geometry. A similar procedure can be implemented for the QC simulator.

Possible effects disturbing only the interaction between the two gravitational field qubit can be implemented in both schemes by varying the relative time arrival of the photons in the probabilistic control-gates.

4.4 Results from the Quantum Circuit simulator

Due to technical[2] difficulties, the experimental simulation is still incomplete. We report the results we obtained, which concern the simulation of the QC simulator.

In the QC simulator implementation, the CNOT gates of the Superposition and Recombination stages are deterministically performed in the single photon path-polarisation space through calcite beam displacers with a fidelity $> 99.5\%$. The Free Fall stage, represented by the Control-Phase gate with a phase equal to π, is performed by the interference of the photon paths at the beam splitter, with a fidelity depending on a number of aspects. First, the fidelity depends on the value of the transmittivity of the beam splitter, which is $|T_H|^2 = 0.329 \pm 0.001$ for the horizontal polarisation and $|T_V|^2 = 0.337 \pm 0.001$ for the vertical polarisation. The fidelity is also affected by the degree of indistinguishability of the interfering photons in all their degrees of freedom. Polarisation, frequency, time of arrival, and spatial mode overlap all affect indistinguishability. Time of arrival and spatial mode overlap are crucial: the arrival time on the BS is controlled by suitable delay lines, while spatial modes are recombined by fine alignment though optical mirrors.

We measured the entanglement witness $\mathcal{W}$ in (4.19), which requires fewer measurements but is not device independent. We obtained a value of $\mathcal{W}^{\text{exp}} = -0.514 \pm 0.002$, violating the separable bound by more than 311 standard deviations. The statistical uncertainty has been computed through a Monte Carlo simulation assuming poissonian statistics.

We performed a CHSH test on the polarisation of the photons at the end of the circuit, obtaining a value $S^{\text{exp}} = 2.401 \pm 0.015$, which violates the classical bound by more than 26 standard deviations.

[2]The experiment had to be put on hold at the beginning of the pandemic. Once experiments could start again, one of the lasers broke. Due to lock-down measures, the technical support staff could not come repair the laser until much later.

4.5 Lessons from the simulations

In the simulations, two systems (the spin qubits) become entangled without interacting directly but by interacting with a third system (the geometry ququart). This mimics closely what happens during the GME experiment, where the spins on the nanodiamonds become entangled with each other while they only couple directly to the position degree of freedom of the masses, which, in turn, couple directly only to the gravitational field. From the logic circuits, it's particularly clear that two non-commuting observables of the geometry ququart come into play, namely $\sigma_x \otimes \sigma_x$ and $\sigma_z \otimes \sigma_z$. When one introduces decoherence in the geometry ququart, these observables commute, and entanglement cannot be created. Another way of putting it is that the geometry ququart starts in an eigenstate of $\sigma_z \otimes \sigma_z$, but coupling to the spins sends it in a superposition of such states. When the superposition is destroyed by decoherence, the spin qubits do not become entangled. Thus the simulations provide a simple illustration of the underlying quantum mechanical principles at play.

On the practical aspect, we expect that completing the simulation will allow to extract useful information about performing the quantum gravity experiment under realistic levels of noise and decoherence. For future research, it would be interesting implementing the simulation for a tunable control-phase gate, which allows to simulate different amounts of free-fall time. It would also be interesting to implement the QC simulator using two pairs of of entangled photons, using non-linear optics to simulate the Recombination stage. However, with current technology, the loss of luminosity is prohibitive.

Chapter 5

Computing the GME phases from first principles

Extant derivations of gravitationally-induced entanglement found in the literature do not include retardation. That is, they neglect any dynamics in the field, and opt to compute things only using a static approximation. While this approximation is quantitatively valid in much of the physical regime of interest, it is conceptually in tension with the crucial assumption that the interaction is mediated by a physical system, rather than being a direct interparticle relation. As we saw in section 2.5, the mediation of the interaction is a central assumption in the no-go theorem that allows to conclude the mediating system—gravity—is behaving non-classically.

In this chapter, based on [60], we develop a Lorentz covariant description of the GME experiment that is thus manifestly local and dynamical. To achieve this, we use the path integral formulation for the metric perturbation of linearised quantum gravity. We then take the stationary phase approximation to compute the quantum phases in the experiment. Path integrals are the appropriate conceptual tool when we want the symmetries of the action to remain explicit. With this strategy, indeed we arrive at a Lorentz covariant description of induced entanglement that is, in addition, invariant under the gauge symmetries of linearised gravity.[1] UV divergences play no role in this regime, thus emphasising the idea that is a prediction of quantum general relativity in this low energy regime. Radiation plays no role in recovering a description of field-induced entanglement that respects all relevant physical principles. This derivation also has technical advantages over previous works. We derive a formula for the phases of arbitrary trajectories. Previous computations assumed that the particles were at rest for most of the experiment so that the phases at the moment of measurement can be approximated as arising during static evolution [34, 164]. Here, we do not need such an assumption, the formulas we derive allow to compute the exact phases given by linearised general relativity for arbitrary particle trajectories, including possibly relativistic trajectories. Finally, the formalism developed here extends to an arbitrary number of particles.

[1] The same can be done in the electromagnetic case, which proceeds analogously and is treated in appendix B.

5.1 Mediated entanglement from the path integral

Let us consider the following experimental setup for observing field induced entanglement. We keep the analysis general by treating the case for n particles, interacting with *some* field $\mathcal{F}$. Later, we will narrow down to the case of two particles.

Assume N particles, each with an embedded spin-1/2 degree of freedom are held in a magnetic trap. Denote by $|\sigma\rangle = \otimes_a |s_a\rangle$, where $s_a \in \{\uparrow, \downarrow\}$, the spin configuration. At time t^{i}, the particles are released and each particle starts from its initial position $\boldsymbol{x}_a^{\mathrm{i}}$ and is put in a spin-dependent planar trajectory $\boldsymbol{x}_a^{s_a}(t)$ by being passed through inhomogeneous and possibly time varying magnetic fields B_z pointing along an axis z. The paths are such that, by the time t^{f}, the particles have returned to their initial positions ($\boldsymbol{x}_a^{\mathrm{f}} = \boldsymbol{x}_a^{\mathrm{i}}$).

If the spin of a particle does not have a definite z component, the magnetic field B_z sets the particle into a path-superposition. The field $\mathcal{F}$ couples to charges q_a of the moving particles. The coupling of B_z with $\mathcal{F}$, the back-reaction of s_a on B_z, and of $\mathcal{F}$ on x_a are taken to be negligible. The spins can become entangled due to their interactions with the field $\mathcal{F}$. At time t^{f} a spin measurement is performed on each particle.

The partition function of the joint system is

$$\mathcal{Z} \approx \int \mathcal{D}\mathcal{F}' \mathcal{D}x' \exp\left(\frac{iS}{\hbar}\right) , \tag{5.1}$$

where $\mathcal{D}x' = \prod_a \mathcal{D}x'_a$, with

$$S = S\left[\boldsymbol{x}'_a(t), \mathcal{F}'(\boldsymbol{x}, t); B_z, \sigma\right] . \tag{5.2}$$

The integration is over field configurations $\mathcal{F}'(\boldsymbol{x}, t)$ and over the paths of the point charges $\boldsymbol{x}'_a(t)$. For the sake of notational simplicity, in what follows, we suppress the dependence of the paths and the field on the space and time coordinates and the dependence of the action on B_z and σ, since they are not affected by the evolution.

The unitary evolution from t^{i} to t^{f} is given by folding the above path integral with initial and final states, yielding the Feynman propagator for the system. We denote such a choice of boundary states as

$$\left|\psi^{\mathrm{i,f}}\right\rangle = \left|F^{\mathrm{i,f}}\left[\boldsymbol{x}_a^{\mathrm{i,f}}\right]\right\rangle \otimes \left|\boldsymbol{x}_a^{\mathrm{i,f}}\right\rangle \tag{5.3}$$

We take that these boundary conditions are the same for all spin configurations σ and that the final time is taken sufficiently in the future, when the field has relaxed and $\langle F^{\mathrm{i}}|F^{\mathrm{f}}\rangle \approx 1$. If it were not the case, the effective tracing out of the field and paths degrees of freedom will hide the field-induced entanglement through false decoherence. In practice, these boundary conditions can approximately taken to be classical static configurations, given by the Newtonian field $F^{\mathrm{i,f}}[\boldsymbol{x}_a^{\mathrm{i,f}}]$ of masses sitting at the initial and final particle positions $\boldsymbol{x}_a^{\mathrm{i,f}}$.

Since the path that each particle takes is determined by the spin and, by assumption, the z-component of the spin does not change along the path, the evolution operator is of the form

$$U_{\mathrm{i}\to\mathrm{f}} = \sum_\sigma |\sigma\rangle\langle\sigma| \otimes U^\sigma_{\mathrm{i}\to\mathrm{f}}, \tag{5.4}$$

where $U^{\sigma}_{\mathrm{i}\to\mathrm{f}}$ is the Feynman propagator between the (identical) boundary states at t^{i} and t^{f} for a given spin configuration σ.

The task now is to compute $U^{\sigma}_{\mathrm{i}\to\mathrm{f}}$. We perform the field integration by taking the stationary phase approximation, keeping only the contribution of field configurations that are on-shell. That is, the action now depends on field configurations $\mathcal{F}[\boldsymbol{x}_a(t)]$ that solve the classical field equations, sourced by particles with trajectories $\boldsymbol{x}_a(t)$ of charge q_a with boundary conditions (5.3). Then,

$$\mathcal{Z} \approx \int \mathcal{D}x' \exp\left(\frac{iS[x'_a, \mathcal{F}[x'_a]\,]}{\hbar}\right). \tag{5.5}$$

This approximation amounts to neglecting the UV pathologies of the field and other divergences. It is thus implicitly assumed that these effects and their proper treatment is not relevant in the low energy regime of interest here. This assumption will be justified *a posteriori*, when nevertheless the procedure yields a complete description of induced entanglement that respects the relevant physical principles of Lorentz invariance and gauge invariance.

During the times t^{i} and t^{f}, for each spin configuration σ there is a classical path $\boldsymbol{x}_a^{s_a}$ determined by the magnetic field B_z coupled to the spin s_a of each particle. These paths can be taken as orthogonal states, and the remaining integral over the paths be approximated by a second stationary phase approximation, keeping only the contribution on these paths. We now have

$$U^{\sigma}_{\mathrm{i}\to\mathrm{f}} \approx \exp\left(\frac{iS^{\mathrm{os}}\left[\boldsymbol{x}_a^{\sigma}, \mathcal{F}[\boldsymbol{x}_a^{\sigma}]\right]}{\hbar}\right)\left|\psi^{\mathrm{f}}\middle\rangle\middle\langle\psi^{\mathrm{i}}\right| \tag{5.6}$$

where the superscript ‘os’ is to remind that S^{os} is the on-shell action for the system.

Let us apply this to a state which is initially in a spin superposition

$$\left|\Psi^{\mathrm{i}}\right\rangle = \sum_{\sigma} A_{\sigma}\left|\sigma\right\rangle \otimes \left|\psi^{\mathrm{i}}\right\rangle \tag{5.7}$$

with A_{σ} complex amplitudes. At the moment of measurement the state is

$$\begin{aligned}\left|\Psi^{\mathrm{f}}\right\rangle &= U_{\mathrm{i}\to\mathrm{f}}\sum_{\sigma} A_{\sigma}\left|\sigma\right\rangle \otimes \left|\psi^{\mathrm{i}}\right\rangle\\ &= \sum_{\sigma} A_{\sigma}\left|\sigma\right\rangle \otimes U^{\sigma}_{\mathrm{i}\to\mathrm{f}}\left|\psi^{\mathrm{i}}\right\rangle\\ \left|\Psi^{\mathrm{f}}\right\rangle &= \sum_{\sigma} A_{\sigma}\exp\left(\frac{iS^{\mathrm{os}}\left[x_a^{\sigma}, \mathcal{F}[x_a^{\sigma}]\right]}{\hbar}\right)\left|\sigma\right\rangle \otimes \left|\psi^{\mathrm{f}}\right\rangle.\end{aligned} \tag{5.8}$$

Notice that in this final state, the spins of the particles are not entangled with the rest of the system any more. However, depending on the values of S^{os}, entanglement can be produced among the spin degrees of freedom.

To make more progress, let’s remind ourselves that the action S in general splits into two parts,

$$S = S_{\mathrm{M}} + S_{\mathcal{F}} \tag{5.9}$$

The term S_{M} does not depend on the field $\mathcal{F}$ and contains the free matter kinetic terms and the coupling of B_z with the spins s_a. The part $S_{\mathcal{F}}$ will contain the kinetic terms for the field $\mathcal{F}$ and the coupling of $\mathcal{F}$ with the charges q_a, but no interparticle interaction. This is the part that describes the *field mediation*. In principle S_{M} can be computed, or measured independently. In practice, it may be preferable to remove the effect of S_{M} by an appropriately symmetric choice of setup, such that $S_{\mathrm{M}}^{\mathrm{os}}$ is the same for all spin configurations and becomes a global phase that can be ignored. Let us assume such a choice of setup has been made. We are then left with

$$\left|\Psi^{\mathrm{f}}\right\rangle \propto \sum_{\sigma} A_{\sigma}\left|\sigma\right\rangle e^{i\phi_{\sigma}} \otimes \left|\psi^{\mathrm{f}}\right\rangle, \tag{5.10}$$

where we have defined

$$\phi_{\sigma} = \frac{S_{\mathcal{F}}^{\mathrm{os}}\left[x_a^{\sigma}, \mathcal{F}[x_a^{\sigma}]\right]}{\hbar} \tag{5.11}$$

The phases ϕ_a are responsible for entanglement production *mediated through* $\mathcal{F}$. They are *manifestly local* because the on-shell action is Lorentz covariant (and gauge invariant). The task at hand now is to calculate the action $S_{\mathcal{F}}^{\mathrm{os}}$.

5.2 The action for the gravitational field of moving particles

In this section we calculate $S_{\mathcal{F}}^{\mathrm{os}}$ when the field $\mathcal{F}$ is the metric perturbation of linearised gravity sourced by point particles. The analogous electromagnetic case is is calculated in appendix B. Since the spacetime curvature is everywhere weak, far smaller than in the vicinity of a black hole horizon, the linearised approximation is applicable. We denote the gravitational perturbation field sourced by the particles as $\mathcal{F}$, which lives on a background Minkowski spacetime.

5.2.1 On-shell action

The action for linearised gravity minimally coupled to the energy-momentum tensor $T^{\mu\nu}$ is given[2] by [161]

$$\begin{aligned} S_{\mathcal{F}} = \frac{c^4}{16\pi G}\int \mathrm{d}^4x\Big(-\frac{1}{4}\partial_{\rho}h_{\mu\nu}\partial^{\rho}h^{\mu\nu} + \frac{1}{2}\partial_{\rho}h_{\mu\nu}\partial^{\nu}h^{\mu\rho} - \frac{1}{2}\partial_{\nu}h^{\mu\nu}\partial_{\mu}h + \frac{1}{4}\partial^{\mu}h\partial_{\mu}h\Big) \\ +\frac{1}{2}\int \mathrm{d}^4x\, h_{\mu\nu}T^{\mu\nu}, \end{aligned} \tag{5.12}$$

where $\mathrm{d}^4x = \mathrm{d}t\mathrm{d}^3x$. The full spacetime metric is given by $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ with the metric perturbation satisfying $|h_{\mu\nu}| \ll 1$, $\eta_{\mu\nu}$ is the Minkowski metric, and $h = \eta_{\mu\nu}h^{\mu\nu}$. The action is invariant under an infinitesimal change of coordinates $x^{\mu} \to x'^{\mu} = x^{\mu} + \xi^{\mu}(x)$ under which the metric perturbation transforms as $h'_{\mu\nu}(x') = h_{\mu\nu}(x) - (\partial_{\mu}\xi_{\nu} + \partial_{\nu}\xi_{\mu})(x)$. We use the gauge freedom to simplify calculations and write the Lagrangian in the harmonic gauge, where the perturbation satisfies

$$\partial^{\mu}h_{\mu\nu} = \frac{1}{2}\partial_{\nu}h. \tag{5.13}$$

[2]Greek indices denote 4–vectors and bold latin letters denote 3–vectors, the metric signature is $(-,+,+,+)$.

Boundary terms at infinity are taken to vanish. In this gauge, the action $S_{\mathcal{F}}$ simplifies to

$$S_{\mathcal{F}} = \frac{c^4}{64\pi G}\int \mathrm{d}^4x\big[-\partial_\rho h_{\mu\nu}\partial^\rho h^{\mu\nu} + \frac{1}{2}\partial^\mu h\partial_\mu h\big] + \frac{1}{2}\int d^4x h_{\mu\nu}T^{\mu\nu} \tag{5.14}$$

after integrating by parts. The Euler-Lagrange equations for $h_{\mu\nu}$ are then

$$\Box h_{\mu\nu} = -\frac{16\pi G}{c^4}\bar{T}_{\mu\nu}, \tag{5.15}$$

where the overbar denotes the operation of trace-reversal:

$$\bar{T}_{\mu\nu} = T_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}T \tag{5.16}$$

with $T = \eta^{\mu\nu}T_{\mu\nu}$. To obtain the on-shell action, we place (5.15) into (5.14) and integrate by parts. We get

$$S_{\mathcal{F}}^{\mathrm{os}} = \frac{1}{4}\int \mathrm{d}^4x\, h_{\mu\nu}T^{\mu\nu}. \tag{5.17}$$

The interaction between matter and gravity is entirely encoded in $S_{\mathcal{F}}$. As shown in the previous section, $S_{\mathcal{F}}^{\mathrm{os}}$ is the central object of interest for observing induced entanglement: this Lorentz covariant and gauge invariant quantity will be measured in the phases of the final spin state. We emphasise that the use of the Lorentz gauge above is an intermediate step in the calculation of a gauge invariant quantity. The same result, requiring more calculations, could be arrived at by using, for instance, a Coulomb-like gauge [209]. Also, note that the on-shell action is simply one half of the coupling term $\frac{1}{2}h_{\mu\nu}T^{\mu\nu}$ of the Lagrangian. This numerical factor is necessary to correctly recover the Newtonian limit (see below).

5.2.2 Point particles

Let us now consider the gravitational interaction of point particles. The use of point particles is an approximation that allows to use an explicit solution of the field equations. One could consider realistic smooth mass distributions and repeat the calculation. The logic and conclusions will be the same. So long as the size of the two matter distributions is much smaller than their separation, the use of point charges will be a good approximation. The stress-energy tensor is

$$T^{\mu\nu}(t,\boldsymbol{x}) = \sum_a m_a\delta^{(3)}(\boldsymbol{x}-\boldsymbol{x}_a(t))V_a^{\mu\nu}(t) \tag{5.18}$$

where

$$V_a^{\mu\nu}(t) = \gamma_a(t)v_a^\mu(t)v_a^\nu(t) \tag{5.19}$$

with $v_a^\mu(t) = (c, d\boldsymbol{x}_a/dt) = (c, \boldsymbol{v}_a)$, where $\boldsymbol{v}_a$ is the three velocity and $\gamma_a(t) = (1-\|\boldsymbol{v}_a(t)\|^2/c^2)^{-1/2}$ the Lorentz factor.

From the equations of motion, the metric perturbation of this mass configuration expressed in the Lorentz gauge is

$$h^{\mu\nu}(t,\boldsymbol{x}) = \frac{4G}{c^4}\sum_a\left[\frac{m_a\bar{V}_a^{\mu\nu}}{d_a - \boldsymbol{d}_a\cdot\boldsymbol{v}_a/c}\right]_{t=t_a(t,\boldsymbol{x})}, \tag{5.20}$$

where $\boldsymbol{d}_a(t,\boldsymbol{x}) = \boldsymbol{x} - \boldsymbol{x}_a(t)$, $d_a = \|\boldsymbol{d}_a\|$ and $\bar{V}_a^{\mu\nu} = V_a^{\mu\nu} - \frac{1}{2}\eta^{\mu\nu}V_a$, $V_a = \eta_{\mu\nu}V_a^{\mu\nu}$. A crucial aspect of the formula above is that all quantities on the right hand side are evaluated at the *retarded* times t_a. The retarded time $t_a = t_a(t,\boldsymbol{x})$ is the time coordinate at which the past lightcone of the event $(t,\boldsymbol{x})$ intersects the trajectory of particle a. It is defined implicitly by the relation:

$$ct - ct_a = \|\boldsymbol{x} - \boldsymbol{x}_a(t_a)\|. \tag{5.21}$$

The solutions (5.20) are the gravitational analogue of the Liénard–Wiechert potentials of electromagnetism [121]. The procedure to derive them is proceeds analogously as in electromagnetism, although they do not appear to be as well known in the literature. Using these solutions, the on-shell action becomes

$$S_{\mathcal{F}}^{\text{os}} = \frac{G}{c^4}\sum_{a,b}^{a\neq b}\int \mathrm{d}t\, \frac{m_a m_b \bar{V}_a^{\mu\nu}(t_{ab})V_{b\mu\nu}(t)}{d_{ab}(t) - \boldsymbol{d}_{ab}(t)\cdot\boldsymbol{v}_a(t_{ab})/c} \tag{5.22}$$

Here, the retarded time $t_{ab} = t_{ab}(t)$ is the coordinate time at which the past lightcone of the event $(t,\boldsymbol{x}_b(t))$ intersects the worldline of particle a. It is defined by the relation

$$ct - ct_{ab} = \|\boldsymbol{x}_b(t) - \boldsymbol{x}_a(t_{ab})\|. \tag{5.23}$$

Then $\boldsymbol{d}_{ab}(t) = \boldsymbol{x}_b(t) - \boldsymbol{x}_a(t_{ab})$ and $d_{ab} = \|\boldsymbol{d}_{ab}\|$.

The quantities given by (5.22) can in principle be computed exactly for any choice of trajectories $\boldsymbol{x}_a$. In the next section, we define some useful approximations.

5.2.3 Three possible approximations, and their relations

There are obvious approximations of interest to the exact expression (5.22) for the on-shell action: the slow-moving approximation and the near-field approximation. These are two distinct approximations. When both approximations are taken, we arrive at the 'Newtonian limit'.

The slow-moving approximation is when the charges or masses are moving with speeds much smaller than the speed of light, $\|\boldsymbol{v}_a\| \ll c$. Since

$$\bar{V}_a^{\mu\nu}V_{b\mu\nu} = c^4 + \mathcal{O}\left(c^3\|\boldsymbol{v}_a\|\right), \tag{5.24}$$

when $\|\boldsymbol{v}_a\| \ll c$, the exact expression (5.22) can be well approximated by

$$S_{\mathcal{F}}^{\text{slow}} = \frac{1}{2}G\sum_{a,b}^{a\neq b}\int \mathrm{d}t\, \frac{m_a m_b}{d_{ab}(t)}. \tag{5.25}$$

Note that $d_{ab}(t)$ still takes into account retardation: it is the distance from the position of particle b at time t to the *retarded* position of particle a. Equation (5.25) can thus be regarded as the retarded version of Newton's law for gravitation. To summarise, the slow-moving approximation retains the finite speed of propagation of signals (via retardation effects) while neglecting other relativistic effects, such as the relativistic corrections to mass and momentum and the effect these have on the field, but includes retardation. In the slow-moving approximation (5.25), the interaction is still *local*.

The second approximation that can be taken for (5.22) is for when the timescale of interest yields a region much larger than the characteristic scale of the spatial separation of the masses (or charges). This can be called the *near-field approximation* in the following sense: if the source's characteristic scale of time variation (divided by c) is much larger than the distance from the other masses, then we can neglect retardation between the masses. In this approximation, the retarded times t_{ab} in (5.22) are replaced by the coordinate time t, yielding an instantaneous interaction

$$S_{\mathcal{F}}^{\text{near}} = \frac{G}{c^4}\sum_{a,b}^{a\neq b}\int \mathrm{d}t \, \frac{m_a m_b \bar{V}_a^{\mu\nu}(t) V_{b\mu\nu}(t)}{\|\boldsymbol{x}_b(t) - \boldsymbol{x}_a(t)\| - (\boldsymbol{x}_b(t) - \boldsymbol{x}_a(t)) \cdot \boldsymbol{v}_a(t)/c}. \tag{5.26}$$

This approximation is clearly distinct from the slow-moving approximation as it disregards retardation but it does not presuppose non-relativistic speeds for the particles.

Lastly, taking *both* of the above approximations yields the Newtonian limit. Indeed, taking equal masses, (5.25) approximates to

$$S_{\mathcal{F}}^{\mathrm{N}} = \int \mathrm{d}t \, \frac{Gm^2}{d(t)}, \tag{5.27}$$

which is the correct result for a Newtonian interaction. It is what one obtains by formally sending $c \to \infty$. Taking d to be constant, we recover the formula used in the literature for the GME [34, 164], where the phase over a period Δt is taken to be given by the Newtonian potential energy multiplied by Δt

$$\frac{S_{\mathcal{F}}^{\mathrm{N}}}{\hbar} = \frac{Gm^2}{d}\frac{\Delta t}{\hbar}. \tag{5.28}$$

5.2.4 Observable effect of retardation

To connect with the GME experiments, let us examine the observable effects of retardation in the simple case of two equal masses. In the experiments, the masses will move at non-relativistic speeds, so that we can take the slow-moving approximation, and spin-dependent phases (5.11) are given by

$$\phi_\sigma = \frac{S_{\mathcal{F}}^{\text{slow}}[x_a^{s_a}]}{\hbar} = \frac{Gm^2}{2\hbar}\int \mathrm{d}t \left(\frac{1}{d_{12}(t)} + \frac{1}{d_{21}(t)}\right). \tag{5.29}$$

This formula will yield some corrections to the Newtonian approximation taken in the original papers [34, 164] and in the rest of the thesis. Besides this *quantitative* effect, *qualitatively* different behaviour can be predicted when the spatial superposition of the particles happens entirely within spacelike separated regions.

The situation is depicted in figure 5.1. Take the particles at rest at a distance d for all times $t < t_1$ and $t > t_2$. Between t_1 and t_2, the particles undergo a spin–dependent motion. The setup is such that $c(t_2 - t_1) < d$, so that the non-stationary parts of the worldlines are spacelike separated. At all times before t_2 and after $t_3 = t_2 + d/c$ then, the retarded position of each particle with respect to the other is constant, meaning that the field is not in superposition there. During the interval from t_2 to

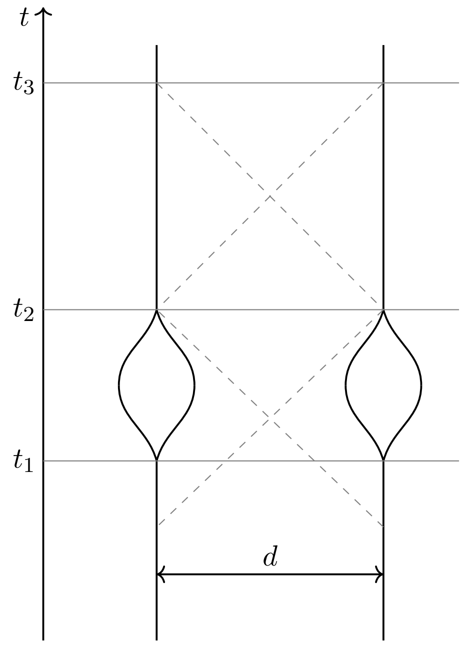


Figure 5.1. Retardation prevents the creation of entanglement. In this setup, the particles are in a superposition in spacelike separated regions. When they are in superposition they move through a static semiclassical field, when the field is in superposition the particles are not. The phases generated in this setup do not lead to entanglement.

t_3, each particles moves in a superposition of fields, but the particles themselves are not in a superposition. In this setup, no entanglement can be generated.

Let $x_a^{s_a}(t)$ be the displacement of particle a from its initial position due to the coupling of the external magnetic field B_z with its spin, so that

$$d_{21}^{\sigma}(t) = d - x_1^{s_1}(t) + x_2^{s_2}(t_{21}). \tag{5.30}$$

Recall that $|\sigma\rangle = \otimes_a |s_a\rangle$. Using (5.25), ϕ_σ is a sum of two integrals that can be done by splitting the domain of integration in four. We have for example

$$\int_{t^{\mathrm{i}}}^{t^{\mathrm{f}}} \frac{\mathrm{d}t}{d_{21}^{\sigma}(t)} = \int_{t^{\mathrm{i}}}^{t_1} \frac{\mathrm{d}t}{d} + \int_{t_1}^{t_2} \frac{\mathrm{d}t}{d - x_1^{s_1}(t)} + \int_{t_2}^{t_3} \frac{\mathrm{d}t}{d + x_2^{s_2}(t_{21}(t))} + \int_{t_3}^{t^{\mathrm{f}}} \frac{\mathrm{d}t}{d}. \tag{5.31}$$

Note that each of these terms depends on only one spin at a time. This implies that the phases can be written as

$$\phi_\sigma = C + \phi_{s_a} + \phi_{s_b}. \tag{5.32}$$

Thus, if the initial states of the spins is separable, so will be the final state. No entanglement is generated. If, on the other hand, one calculates the phase in the Newtonian limit with instantaneous interaction, the masses and hence the spins result in an entangled state.

This effect can in principle be observed, although it would certainly be easier to do with electric charge and electromagnetic interactions.

5.3 Conclusion

We have computed the phases giving rise to gravity mediated entanglement and shown that they are manifestly Lorentz invariant (thus causal) and gauge invariant quantities. We derived the approximation when the particle motion is non-relativistic and showed that this is still local as it includes the corrections for retardation. Finally, we have seen that retardation has an observable effect in the production of induced entanglement.

Part III

About Time

Chapter 6

An experiment to test the discreteness of time

Optical clocks using strontium ^{87}Sr are among the most accurate in the world. The time elapsed between two of their ticks is about 10^{-15} s (the inverse of strontium frequency) with a precision of 10^{-19} [168]. Physical phenomena that probe much smaller characteristic timescales have also been measured. For instance, the lifetime of the top quark is 10^{-25} s. Such a result is obtained experimentally from a statistical analysis, where the short duration of the lifetime is compensated by a large number of events. Theoretical physics features even shorter scales: in primordial cosmology, the inflation epoch is believed to have lasted 10^{-32} s. Based on a cosmological model, the recent paper [260] even argues that the precision of recent atomic clocks already sets an upper bound of 10^{-33} s for a fundamental period of time.

Planck time is a far smaller timescale. We recall that the planck time is defined as

$$t_{\mathrm{P}} \stackrel{\mathrm{def}}{=} \sqrt{\frac{G\hbar}{c^5}} \approx 10^{-44} \text{ s}, \tag{6.1}$$

where G is Newton's constant, $\hbar$ the reduced planck's constant and c the speed of light. It can seem an impossible task to probe time at the planck scale. However, the example of the lifetime of the top quark shows that it is possible to overtake clock accuracy limitations by several orders of magnitude using statistics. In this chapter, we examine the following question: if time behaves differently from a continuous variable at the planckian scale, how could the departure from this behaviour be inferred experimentally? To answer this question, we assume that proper time differences take discrete values in multiple steps of planck time, and devise a low energy experiment that would detect this effect.

The proposal in this chapter is motivated by the recent experimental proposals to detect the non-classicality of the gravitational field by detecting gravity mediated entanglement (GME) [33, 34, 149, 164, 166] and the production of non-gaussianity [140] introduced in chapter 1. Since probing the quantum gravity regime with particle colliders may be practically impossible, it is intriguing that these low energy experiments are not too far removed from current capabilities. Instead of accelerators, the suggestion in these proposals is to quantum control slow moving nanoparticles or use a Bose-Einstein condensate.

Here, we will see that a similar setup, with similar technological requirements, is able to probe planck-sized time intervals The plan is the following. In section 6.1, we present the experimental setup, which involves a single mass going through a matter-wave interferometer next to a source mass. The two masses interact only via gravity. In section 6.2, we introduce the hypothesis that proper time differences are discrete at the planck level, and discuss how this affects the measurements at the end of the interferometer. Detecting these effects imposes a number of constraints on the experimental parameters, which we deduce in section 6.3. Then, in section 6.4, we suggest a set of reasonable parameters that fulfil these constraints. In section 6.5, we complete the analysis by checking that it is possible to keep environmental decoherence at bay long enough. Finally, in section 6.6 we discuss the time-discreteness hypothesis itself. This chapter is based on [61].

6.1 Experimental setup

The proposed experimental setup is depicted in figure 6.1. A spherical nanoparticle of mass m with embedded magnetic spin is dropped simultaneously with a second mass M. The mass m is then put into a spin-dependent superposition of paths by the application of a series of electromagnetic pulses. This technique was proposed in [34, 265]. In the branch of closest approach, m and M are at a distance d, in the other, they are at a distance $d + l$. The superposition is held at these distances for a time t, as measured in the laboratory frame. While the two masses free fall, they interact gravitationally. If linearised quantum gravity holds, then the two quantum branches in the total state evolve differently, accumulating a relative phase. After the superposition has been undone, this phase is visible in the state of the spin of the mass m.

Let us see this in detail. The quantum state of the mass m is given by its position in the apparatus and the orientation of its embedded spin. There will be three relevant position states[1] $|L\rangle$, $|C\rangle$ and $|R\rangle$, respectively left, centre and right. For the spin, we use the canonical basis, $|\uparrow\rangle$ and $|\downarrow\rangle$, in the z-direction. The mass m is prepared at t_0 in the central position with the spin in the positive x-direction:

$$|\psi_0\rangle = \frac{1}{\sqrt{2}}|C\rangle\left(|\uparrow\rangle + |\downarrow\rangle\right). \tag{6.2}$$

An inhomogeneous magnetic field is then applied to the mass m, entangling its position with its spin so that at time t_1 the state is

$$|\psi_1\rangle = \frac{1}{\sqrt{2}}\,|L\uparrow\rangle + \frac{1}{\sqrt{2}}\,|R\downarrow\rangle. \tag{6.3}$$

The particle is then allowed to free-fall for a time t. During this time, it interacts gravitationally with the mass M. The displacement of the masses due to their gravitational attraction is negligible. The two states $|L\rangle$ and $|R\rangle$ are eigenstates of the hamiltonian and each acquires a phase proportional to the newtonian potential

[1] It has recently been shown [54] that treating the position states as eigenstates is a valid approximation in this setup.

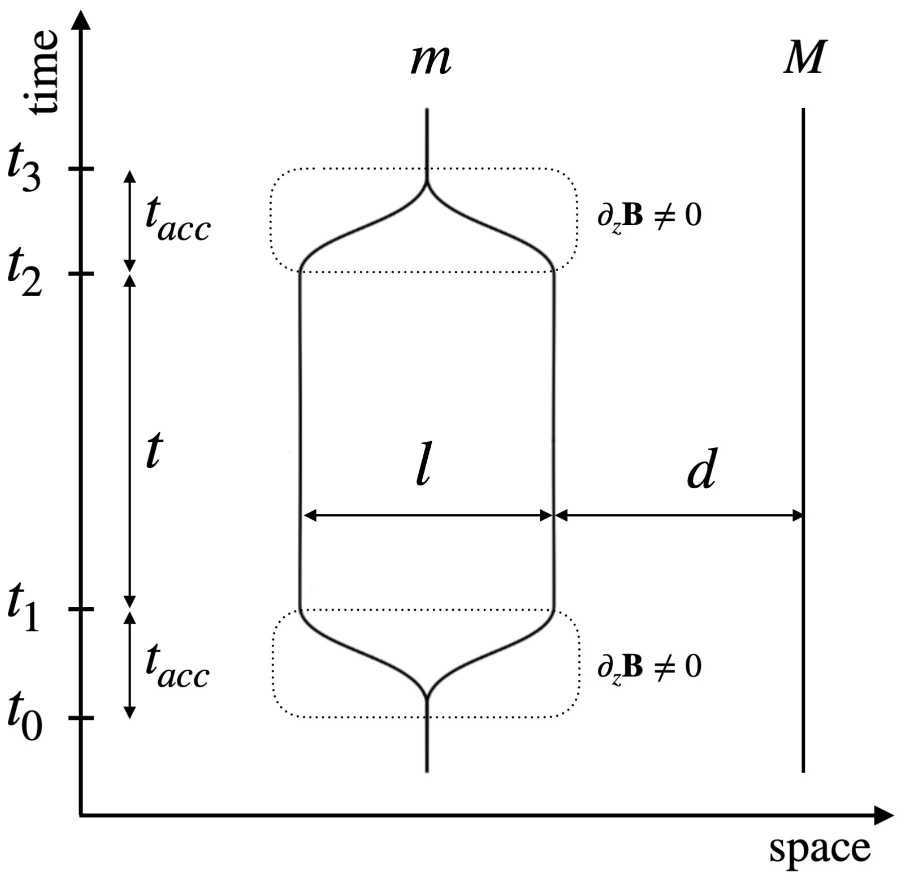


Figure 6.1. Spacetime view of the experiment. For a time t_{acc}, an inhomogeneous magnetic field is applied that sets a mass m with embedded spin in a superposition of two paths, at a distance d and $d + l$, respectively, from another mass M. The masses are in free fall for a time t, as measured in the laboratory, after which the procedure is reversed and the superposition undone. During this time t, the two trajectories accumulate a different phase due to the gravitational interaction with M.

induced by M. So at time t_2 the state is

$$|\psi_2\rangle = \frac{1}{\sqrt{2}} e^{i\phi_L} |L\uparrow\rangle + \frac{1}{\sqrt{2}} e^{i\phi_R} |R\downarrow\rangle \,, \tag{6.4}$$

where

$$\phi_L = \frac{GMm}{\hbar} \frac{t}{d+l} \qquad \text{and} \qquad \phi_R = \frac{GMm}{\hbar} \frac{t}{d}. \tag{6.5}$$

At this point, another inhomogeneous magnetic field is applied to undo the superposition. The final state of the particle is, up to a global phase,

$$|\psi_3\rangle = \frac{1}{\sqrt{2}} |C\rangle \left(|\uparrow\rangle + e^{i\delta\phi} |\downarrow\rangle \right) , \tag{6.6}$$

where the relative phase $\delta\phi$ is given by

$$\delta\phi = \frac{GMmt}{\hbar} \frac{l}{d(d+l)}. \tag{6.7}$$

Information about the gravitational field is now contained in the state of the spin, which in turn can be estimated from the statistics of spin measurements. Concretely, we consider a measurement on the spin of the particle along the y-direction

$$|\pm i\rangle = \frac{1}{\sqrt{2}}|\uparrow\rangle \pm i\frac{1}{\sqrt{2}}|\downarrow\rangle . \tag{6.8}$$

Born's rule gives the probability P_+ of finding the spin in the state $|+i\rangle$:

$$P_+(m, M, d, l, t) = \frac{1}{2} + \frac{1}{2}\sin\delta\phi, \tag{6.9}$$

where we compute $\delta\phi$ as a function of m, M, d, l and t through equation (6.7). This equation for the probability is a theoretical prediction of linearised quantum gravity.

Experimentally, the probability can be measured by the relative frequencies in collected statistics. The experiment is repeated N times keeping the experimental parameters fixed. If the outcome $|+i\rangle$ is recorded N_+ times, the frequency

$$p_+(m, M, d, l, t) = \frac{N_+}{N} \tag{6.10}$$

is then the experimentally measured value of the probability. This procedure can be repeated for different sets of experimental parameters to verify the functional dependence of p_+ to these. In what follows, we propose an experiment that can detect a statistically significant discrepancy between P_+ and p_+. This discrepancy would signal a departure from linearised quantum gravity.

The above experimental setup is similar to that proposed to detect GME in [34], with the main difference that for our purpose we only require one mass, not two, in a superposition of paths. It is thus conceptually more similar to the celebrated Colella-Overhauser-Werner (COW) experiment [3, 69]. However, the task we have set ourselves here and the method to achieve it, goes much beyond showing that gravity can affect a quantum mechanical phase and induce an interference pattern. To detect a potential discreteness of time, we need a more sensitive apparatus, and so the gravitational source M will need to be much *weaker*. In our case, M is not the Earth, but a mesoscopic particle, essentially a speck of dust.

6.2 Hypothesis: time discreteness

While the newtonian limit of linearised quantum gravity is sufficient to compute the phase difference $\delta\phi$, it is can also be understood in general relativistic terms [62, 63]. The mass M induces a Schwarzschild metric which dilates time differently along each of the two possible trajectories of m. Then, equation (6.7) can be recast as

$$\delta\phi = \frac{m}{m_{\mathrm{P}}}\frac{\delta\tau}{t_{\mathrm{P}}}, \tag{6.11}$$

where $\delta\tau$ is the difference of proper time between the two trajectories, given by

$$\delta\tau = \frac{GM}{c^2}\frac{l}{d(d+l)}t. \tag{6.12}$$

Now, it is widely believed that the smooth geometry of general relativity should be replaced, once quantised, by some discrete structure. In particular, we may expect time to be granular in some sense. In which sense precisely, we do not know. However, since $\delta\tau$ admits a straightforward interpretation of a covariant quantum clock, it makes a good candidate to reveal discrete features of time. Thus we make the following hypothesis: $\delta\tau$ can only take values which are integer multiples of planck time t_{P}. That is, (6.12) is modified to:

$$\delta\tau = n\, t_{\mathrm{P}}, \quad n \in \mathbb{N}. \tag{6.13}$$

Additional motivation for the hypothesis and possible alternatives are discussed in section 6.6. For now, it can be taken just as the simplest implementation of the idea that time is discrete at a fundamental level, similar in philosophy to the idea that everyday-life matter is not continuous, but instead made of atoms. Devising an experiment to detect this discreteness and examining its feasibility is the task we have set ourselves in this work.

Equation (6.13) is still incomplete and we need to posit a functional relation between the level n and the parameters M, d, l, t. We rewrite equation (6.12) as

$$\delta\tau = \frac{t}{\beta}\, t_{\mathrm{P}}, \tag{6.14}$$

where we have defined

$$\beta = \frac{d(d+l)c^2}{GMl} t_{\mathrm{P}}. \tag{6.15}$$

Note that β has dimensions of time. To make the hypothesis quantitative, we take n to be given by the floor function[2]

$$n = \left\lfloor \frac{t}{\beta} \right\rfloor. \tag{6.16}$$

That is, n is the integer part of the dimensionless quantity t/β. The main lessons of our results do not depend on the specific choice (6.16) for the functional dependence between t/β and n. Other modifications of the continuous behaviour in (6.12), so long as they display features of planckian size, could be probed by the experiment.

Thus, we have

$$\delta\tau = \left\lfloor \frac{t}{\beta} \right\rfloor t_{\mathrm{P}}. \tag{6.17}$$

The consequences of this hypothesis are revealed in the measured probability p_+ of equation (6.10). If time behaves continuously, p_+, as a function of time t/β will fit the smooth (blue) curve in figure 6.2, given by

$$P_+ = \frac{1}{2} + \frac{1}{2}\sin\left(\frac{m}{m_{\mathrm{P}}}\frac{t}{\beta}\right). \tag{6.18}$$

If the hypothesis holds, the observed profile for the probability will follow that of the orange step function in figure 6.2, given by

$$P_+^{\mathrm{h}} = \frac{1}{2} + \frac{1}{2}\sin\left(\frac{m}{m_{\mathrm{P}}}\left\lfloor \frac{t}{\beta} \right\rfloor\right). \tag{6.19}$$

[2] The *floor* $\lfloor x \rfloor$ of a real number x is the largest integer smaller than x, aka the *integer part* of x.

To test the hypothesis, the strategy is thus to plot experimentally the curve $p_+(t/\beta)$. Observing plateaux would be the signature of time-discreteness.

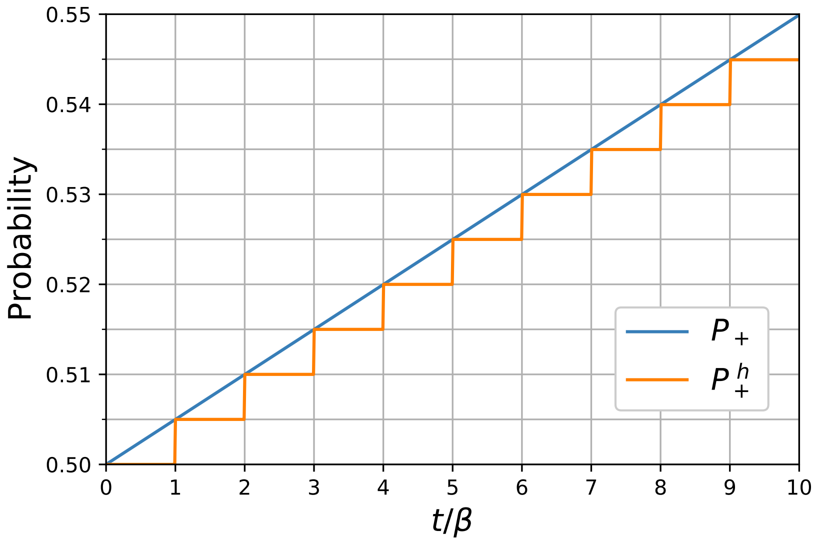


Figure 6.2. Probability of measuring spin $|{+i}\rangle$ as a function of t/β under the continuous and discrete time hypotheses. Blue line: $\delta\tau$ is smooth as in equation (6.14). Orange line: $\delta\tau$ is discrete as in equation (6.17). We have taken the value of $m = 10^{-2} m_{\mathrm{P}}$. The experimental parameters shown in table 6.1 would produce 100 data points scanning the range of t/β depicted here, with a sufficient resolution to decide which of the two curves is realised in nature.

6.3 Ensuring visibility of the effect

Each experimental data point for $p_+(t/\beta)$ is obtained from computing the statistical frequency of the outcome $|{+i}\rangle$. Point by point, a scatter plot of p_+ against t/β will be obtained. We must choose the experimental parameters so that the difference between P_+ and P_+^{h} can be resolved. This imposes requirements on the minimal precision of the experimental apparatus and on the maximal permissible gravitational noise in the environment.

6.3.1 Visibility of the vertical axis

The uncertainty Δp_+ for the probability p_+ after N runs results from using finite statistics and is of the order

$$\Delta p_+ \sim \frac{1}{\sqrt{N}}. \tag{6.20}$$

The vertical step α between the plateaux is given by

$$\alpha = \left| \sin\left(\left(\left\lfloor \frac{t}{\beta} \right\rfloor + 1 \right) \frac{m}{m_{\mathrm{P}}} \right) - \sin\left(\left\lfloor \frac{t}{\beta} \right\rfloor \frac{m}{m_{\mathrm{P}}} \right) \right|. \tag{6.21}$$

We assume that $m \ll m_{\mathrm{P}}$, consistent with the fact that it is hard to put a large mass in a superposition. The above expression simplifies to

$$\alpha(t) \approx \frac{m}{m_{\mathrm{P}}} \cos\left(\left\lfloor \frac{t}{\beta} \right\rfloor \frac{m}{m_{\mathrm{P}}}\right). \tag{6.22}$$

So the steps are most visible when

$$\left|\frac{t}{\beta}\frac{m}{m_{\mathrm{P}}}\right| \ll 1. \tag{6.23}$$

Then the expression simplifies to

$$\alpha(t) \approx \frac{m}{m_{\mathrm{P}}}. \tag{6.24}$$

Requiring that the probability uncertainty is an order of magnitude smaller than the vertical step, $\Delta p_+ < 10^{-1}\alpha$, we find the constraint

$$N > 10^2 \left(\frac{m_{\mathrm{P}}}{m}\right)^2. \tag{6.25}$$

We see that a larger mass m means that fewer runs N per data point are required, which implies a shorter total duration T_{tot} of the experiment. Indeed, since plotting $p_+(t/\beta)$ requires N runs per data point, each run requiring at least a time t, a lower bound for the total duration of the experiment is

$$T_{\mathrm{tot}} \sim N_{\mathrm{dp}} N t, \tag{6.26}$$

where N_{dp} is the number of data points. Thus, the constraint (6.25) can be restated as

$$\frac{T_{\mathrm{tot}}}{N_{\mathrm{dp}}\, t} > 10^2 \left(\frac{m_{\mathrm{P}}}{m}\right)^2. \tag{6.27}$$

This constraint imposes a trade-off between the time required to resolve the discreteness and the mass that has to be in superposition. It counter-balances the fact that it is harder to achieve quantum control of a large mass.

6.3.2 Visibility of the horizontal axis

The uncertainty in t/β is found via the standard formula for the propagation of uncertainty and can be expressed as

$$\Delta(t/\beta) = U \frac{t}{\beta}, \tag{6.28}$$

where

$$U \stackrel{\text{def}}{=} \sqrt{\left(\frac{\Delta t}{t}\right)^2 + \left(\frac{d}{d+l} + 1\right)^2 \left(\frac{\Delta d}{d}\right)^2 + \left(\frac{\Delta M}{M}\right)^2 + \left(\frac{d}{d+l}\right)^2 \left(\frac{\Delta l}{l}\right)^2}. \tag{6.29}$$

By assumption (6.16), the width of the plateaux is 1. To place several data points on each plateau, we require the typical uncertainty to be an order of magnitude smaller, *i.e.* $\Delta(t/\beta) < 10^{-1}$. We thus impose the constraint

$$U < 10^{-1}\, \frac{\beta}{t} \tag{6.30}$$

on the experimental parameters. Note that a given U determines the highest value of $n = \lfloor t/\beta \rfloor$ for which the discontinuities can be resolved.

6.3.3 Gravitational noise

There is no analog of a Faraday cage for gravitational interactions, so influences by other masses will also contribute to the accumulated phase $\delta\phi$. Since the experiment we are considering is in a sense an extremely sensitive gravimeter, these would need to be taken carefully into account. We distinguish between 'predictable' gravitational influences and 'unpredictable' gravitational influences, *i.e.* gravitational noise. The latter type will dictate the degree of isolation required for a successful realisation of the experiment, adding another visibility constraint, while the former type can be dealt with by calibration.

The presence of unexpected masses in the vicinity of the apparatus may disturb the measurement. It will contribute to the proper time dilation by an amount η, modifying (6.19) to

$$P_+^{\mathrm{h}}(\eta) = \frac{1}{2} + \frac{1}{2}\sin\left(\frac{m}{m_{\mathrm{P}}}\left\lfloor\frac{t}{\beta} + \frac{\eta}{t_{\mathrm{P}}}\right\rfloor\right). \tag{6.31}$$

Getting a single data-point requires N drops, and for each drop, the perturbation η may be a priori different. However, it should be small enough so that it does not make the probability P_+^{h} jump to another step, *i.e.* η is a negligible noise if

$$\left\lfloor\frac{t}{\beta} + \frac{\eta}{t_{\mathrm{P}}}\right\rfloor = \left\lfloor\frac{t}{\beta}\right\rfloor. \tag{6.32}$$

Of course, η is a random variable over which the control is limited. To a first approximation, the condition (6.32) can be implemented over the N drops by requiring

$$\Delta\eta < 10^{-1} t_{\mathrm{P}}. \tag{6.33}$$

For instance, the gravitational noise induced by the presence of a mass μ at a distance $D \gg l, d$ is at most

$$\eta_{\max} = \pm\frac{G\mu l}{D^2}\frac{t}{\hbar}. \tag{6.34}$$

Thus, we get a fair idea of how isolated the apparatus should be with the condition

$$2Gl\frac{\mu}{D^2}\frac{t}{\hbar} < 10^{-1} t_{\mathrm{P}}. \tag{6.35}$$

The ratio

$$A = \frac{\mu}{D^2} \tag{6.36}$$

is a measure of the impact that a mass μ has on the visibility of the discontinuities, if it is allowed to move uncontrollably as close as a distance D away from the experiment. Thus, we end up with the following constraint

$$A\,l\,t < 5\times 10^{-2}\,\frac{t_{\mathrm{P}} m_{\mathrm{P}}}{l_{\mathrm{P}}}. \tag{6.37}$$

This equation is a requirement on the control of the environment necessary to resolve the discontinuities. All things being equal, shorter superpositions are less sensitive to the gravitational noise.

Above, we took into account the effect of a single mass μ. This not sufficient to guarantee that there will not be a cumulative effect from several masses around. However, note that if these masses are homogeneously distributed, their contributions may average out.

The 'predictable' type of gravitational influences are systematic errors arising, for example, from the gravitational field of the Earth, the Moon, and the motion of other large bodies, such as tectonic activity or sea tides, but also from small masses that will unavoidably be present in the immediate vicinity of the mass m, such as the experimental apparatus itself and the surrounding laboratory. Given the extreme sensitivity of the apparatus, it will likely not be possible to make all these gravitational influences satisfy (6.37). However one can calibrate for the contribution of a mass μ at distance D if it moves slowly with respect to the time Nt that it takes to collect a data point *i.e.* if

$$Ntv \ll D, \tag{6.38}$$

with v the speed of the mass. Another possibility that can be calibrated for is if the mass is not moving slowly but the uncertainty in its position is small with respect to D (for instance, a moving mechanical part or the Moon).

An example of a calibration procedure is as follows. Let us assume that the different values of t/β are obtained by changing d while keeping M, l, and t fixed (as considered in the next section). The mass μ will contribute a constant phase ϕ_B, which we can estimate by running the experiment without M. So long as the masses are slow moving, it suffices to rotate the measurement basis to

$$\frac{1}{\sqrt{2}}\left|0\right\rangle + \frac{1}{\sqrt{2}} e^{i\left(\phi_B \pm \frac{\pi}{2}\right)}\left|1\right\rangle \tag{6.39}$$

rather than $\{|\pm i\rangle\}$.

6.4 Balancing act

Let us summarise the three experimental constraints that we derived above.

$$\begin{cases} 10^2\,\dfrac{N_{\mathrm{dp}}\,t}{T_{\mathrm{tot}}} < \left(\dfrac{m}{m_{\mathrm{P}}}\right)^2 & \text{[vertical]} \\ U\dfrac{t}{\beta} < 10^{-1} & \text{[horizontal]} \\ A\,l\,t < 5\times 10^{-2}\,\dfrac{t_{\mathrm{P}}\,m_{\mathrm{P}}}{l_{\mathrm{P}}} & \text{[noise],} \end{cases} \tag{6.40}$$

with

$$\frac{t}{\beta} = \frac{M}{m_{\mathrm{P}}}\frac{ctl}{d(d+l)}. \tag{6.41}$$

These constraints have to be satisfied in order to have a chance to resolve the discontinuities in p_+ that would result from our hypothesis. We now proceed to identify a set of reasonable parameters that satisfy the constraints. We will make a series of assumptions based on current technological trends. We will number the assumptions to make them visible.

1. Any of the parameters M, d, l and t could be modulated to scan a range of t/β. Since t/β is most sensitive to changes in d (quadratic dependence), we assume the modulation of d, keeping M, l and t fixed.

2. The total duration of the experiment is about a year
$$T_{\text{tot}} \sim 10^7 \text{ s.} \tag{6.42}$$

3. The plot requires about a hundred data points
$$N_{\text{dp}} \sim 10^2, \tag{6.43}$$
to be distributed over ten plateaux
$$t/\beta \leq 10. \tag{6.44}$$

4. Experimentally, the maximal distance between the two branches of the superposition cannot be very large, and so we assume
$$d \gg l. \tag{6.45}$$

From these first assumptions, the system of inequalities (6.40) simplifies to

$$\begin{cases} t < 10^3 \left(\dfrac{m_{\text{P}}}{m}\right)^2 \text{ s} & \text{[vertical]} \\ U < 10^{-2} & \text{[horizontal]} \\ A\, l\, t < 5 \times 10^{-2}\, \dfrac{t_{\text{P}}\, m_{\text{P}}}{l_{\text{P}}} & \text{[noise]} \\ t/\beta \leq 10 & \text{[range],} \end{cases} \tag{6.46}$$

with
$$\frac{t}{\beta} = \frac{M}{m_{\text{P}}} \frac{ctl}{d^2}. \tag{6.47}$$

The uncertainty U, defined by equation (6.29), depends on the precision in t, M, d and l. With the assumption $l \gg d$ its expression simplifies to

$$U = \sqrt{\left(\frac{\Delta t}{t}\right)^2 + \left(\frac{\Delta M}{M}\right)^2 + \left(\frac{\Delta d}{d}\right)^2 + \left(\frac{\Delta l}{l}\right)^2}. \tag{6.48}$$

Then, the [horizontal] inequality implies that each of t, M, d, and l will have to be controlled better than 1 part in 100.

5. It is reasonable to expect that the uncertainty U will be dominated by the uncertainty in the superposition size l, thus,
$$U \approx \frac{\Delta l}{l}. \tag{6.49}$$

6. We assume possible to control the size of the superposition to the scale of a few atoms, i.e.
$$\Delta l = 10^{-9} \text{ m.} \tag{6.50}$$

7. From the above two points we have a lower bound for the value of l. Taking l larger, would only make the experiment harder because of decoherence and gravitational noise. We thus take
$$l \sim 10^{-7}\ \mathrm{m}, \tag{6.51}$$
which satisfies the [horizontal] constraint, allowing to resolve the first 10 steps.

We have now solved the [horizontal] constraint and fixed l. The remaining constraints evaluate to
$$\begin{cases} t < 10^3 \left(\dfrac{m}{m_\mathrm{P}}\right)^2 \mathrm{s} & \text{[vertical]} \\ A\,t < 4\times 10^{-11}\ \mathrm{kg\ s\ m^{-2}} & \text{[noise]} \\ \dfrac{Mt}{d^2} < 7\times 10^{-9}\ \mathrm{kg\ s\ m^{-2}} & \text{[range].} \end{cases} \tag{6.52}$$
All three equations suggest to take t as small as possible. Nonetheless, this cannot be too short because the superposition is created by a magnetic field B that separates the branches at a distance l. This process requires some time t_acc, which is bounded from below by the highest magnetic field B_max that can be created in the lab. Concretely[3]
$$\mu_\mathrm{B}\frac{B_\mathrm{max}}{l} > \frac{ml}{t_\mathrm{acc}^2}, \tag{6.53}$$
where μ_B is the Bohr magneton ($\mu_\mathrm{B} \approx 10^{-23}$ J.T^{-1}).

8. t should be at least as long as t_acc, say
$$t \sim 3\,t_\mathrm{acc}. \tag{6.54}$$

9. Taking $B_\mathrm{max} \sim 10^2$ T, which is the value of the strongest pulsed non-destructive magnetic field regularly used in research [150], we get, in SI units,
$$10^{-8}t^2 > m. \tag{6.55}$$

10. Considering the difficulty to put a heavy mass in superposition, we can minimise both t and m under the [vertical] constraint of (6.52) and equation (6.55). We find
$$\begin{aligned} m &= 3\cdot 10^{-10}\ \mathrm{kg} \sim 10^{-2} m_\mathrm{P} \\ t &= 10^{-1}\ \mathrm{s}. \end{aligned} \tag{6.56}$$

These values are consistent with the assumptions made above that $m \ll m_\mathrm{P}$ and $\Delta t/t \ll 10^{-2}$. We have thus solved the [vertical] constraint too. We are left with
$$\begin{cases} A < 4\times 10^{-10}\ \mathrm{kg\ m^{-2}} & \text{[noise]} \\ \dfrac{M}{d^2} < 7\times 10^{-8}\ \mathrm{kg\ m^{-2}} & \text{[range].} \end{cases} \tag{6.57}$$

[3] We assume the masses are made of a material that allows neglecting diamagnetic effects. If diamagnetism cannot be ignored, one has to resort to a more complicated scheme of pulses, inverting the direction of the magnetic field gradient at specific intervals as detailed in [197], or inverting both the direction of the gradient and the spins as proposed in [256]. Alternatively, one can use a different method of wavepacket separation, like that detailed in [208].

11. Considering the *a priori* difficulty to isolate the system from external perturbations, the noise inequality fixes the minimal upper bound for A, *i.e.* we want to tolerate perturbations as high as

$$A = 4 \times 10^{-10} \text{ kg m}^{-2}. \tag{6.58}$$

This threshold is very sensitive. To give an example, it corresponds to the gravity induced by a bee flying 230m away. Such a high control might only be attainable in space, where cosmic dust particles, with typical mass of 5μg [45], would need to be kept 4 m away from the masses.

We are thus left with one last inequality which reads, in SI units,

$$d > 4 \times 10^3 \sqrt{M}. \quad \text{[range]} \tag{6.59}$$

12. We have implicitly assumed that m is a test mass moving in the geometry defined by M, so we require $M \gtrsim 10\ m$ for consistency. Choosing the minimal value

$$M = 10\ m, \tag{6.60}$$

leads to

$$d \geq 0.17 \text{ m}. \tag{6.61}$$

This corresponds to the lower bound for the range that d will scan, corresponding to $t/\beta = 10$. The value $t/\beta = 1$ provides an upper bound of $d \approx 54$ cm. Note that the assumption made above that $\Delta d/d$, $\Delta M/M \ll 10^{-2}$ is indeed reasonable.

Casimir-Polder. So far, we have not taken into account the Casimir-Polder (CP) force between the two masses. The modification of the vacuum energy between two perfectly conducting, parallel discs of area a a distance d apart [236] results in a force $F_{\text{CP}} = \frac{\hbar c \pi^2}{240 d^4} a$. Taking this force as an overestimate of that between two spherical dielectric particles of cross-sectional area a a distance d apart, we see that the CP force is at most a million times weaker than the gravitational force and can thus be neglected.

Uncertainty on m. A small shift δm on the mass m adds a phase difference $\epsilon = \delta m/m_{\text{P}} \cdot \lfloor t/\beta \rfloor$, which in turn causes a shift δP in the probability. Since $m \ll m_{\text{P}}$ and $t/\beta < 10$, then $\epsilon \ll 1$ and the shift is to first order $\delta P \approx \frac{1}{2}\epsilon$. The uncertainty in m does not affect the visibility of the probability axis if $\delta P \ll \alpha$, *i.e.* if $\delta m/m \ll 2/\lfloor t/\beta \rfloor$. This last condition on m means that the mass m should be known to one part in 100, which is easily reachable.

This concludes our derivation of a set of parameters that satisfies the constraints of the previous section and, thus, allows to probe planckian features of time. The values are summarised in table 6.1. As a corroboration of the analysis, the experimental plot is simulated for these parameters in figures 6.3 and 6.4. There, we see how the effect becomes visible when the gravitational noise and the uncertainty on the experimental parameters satisfy the constraints derived above.

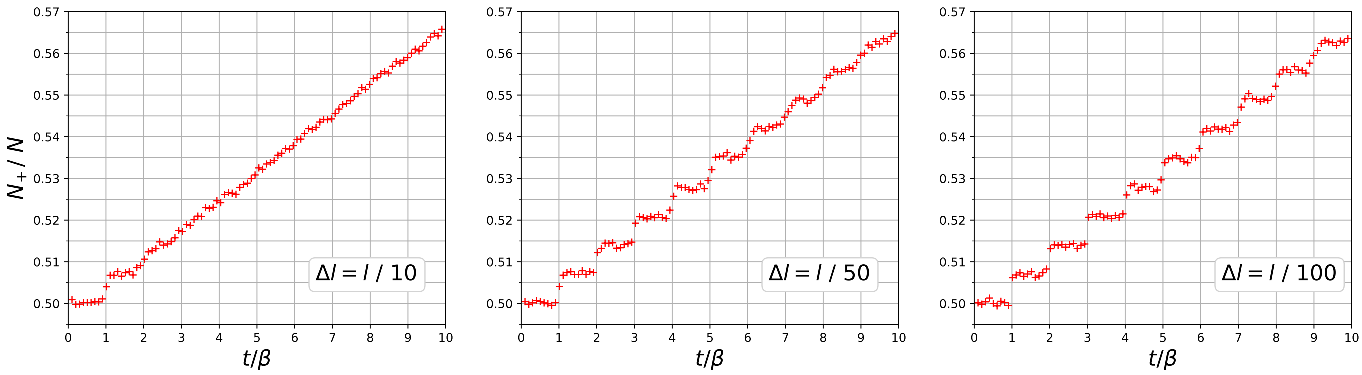


Figure 6.3. Simulated data points with decreasing values of Δl. The value of the parameters is set as in table 6.1, assuming no gravitational noise. Each point point is obtained by sampling N times the probability distribution P_+^{ph} in (6.19), where the parameters t , l and d are themselves each time sampled from a normal distribution with the corresponding uncertainty. From left to right, the uncertainty in l takes the values 10^{-8}m, 5×10^{-9}m and 10^{-9}m, demonstrating that the effect becomes visible when the experimental parameters have little uncertainty, see section 6.3.2. Note how the discontinuities on higher values of t/β require higher precision to be resolved.

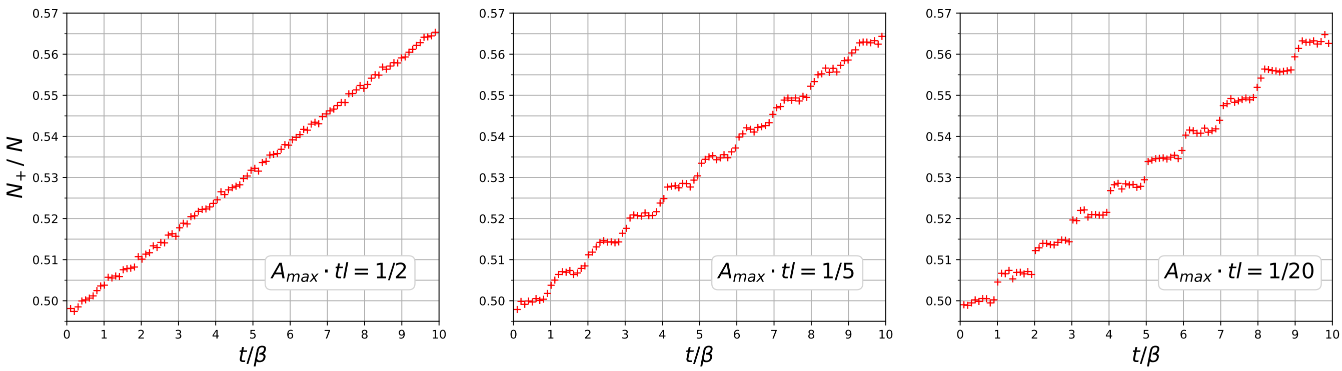


Figure 6.4. Simulated data points with decreasing gravitational noise. The data points are obtained in the same manner as those in figure 6.3, with the following difference. At each run, a value of A is picked uniformly at random from $[-A_{max}, A_{max}]$ and the quantity Alt is added to t/β before sampling the distribution. This procedure simulates the influence of a single mass moving uncontrollably while statistics are collected, see section 6.3.3. The value of the parameters is as set in table 6.1, while A_{max} is, from left to right, $1/(2tl), 1/(5tl)$ and $1/(20tl)$ in natural units. The discontinuities become visible only if the gravitational noise is reduced.

Parameter	Value	Uncertainty
m	3×10^{-10} kg	10^{-12} kg
M	3×10^{-9} kg	10^{-11} kg
t	10^{-1} s	10^{-4} s
l	10^{-7} m	10^{-9} m
d	$[17, 54]$ cm	10^{-2} cm
A	$\leq 4 \times 10^{-10}$ kg m^{-2}	
N_{dp}	100	
N	10^6	
T_{tot}	1 year	
n	$[0, 10]$	

Table 6.1. The experimental parameters identified in section 6.4

6.5 Maintaining coherence

We saw in section 2.3 that a mass in superposition of paths will interact with the ambient black body radiation and stray gas molecules in the imperfect vacuum of the device. As the photons and molecules get entangled with the position degrees of freedom of the mass, the coherence of the superposition is lost and the phase cannot be recovered by observing interference between the two paths. These unavoidable environmental sources of decoherence are well studied both theoretically and experimentally [208, 217, 218]. Gravitational time dilation can also be a source of decoherence for thermal systems [207], but requires much stronger gravitational fields than considered in this experiment.

We assume the experiment will be performed with a nanoparticle of mass $m = 3 \times 10^{-10}$ kg, radius $R = 30\ \mu$m. For the formulas appearing in this section we refer the reader to [208].

Black-body radiation

The typical wavelength of thermal photons ($\approx 10^{-5}$ m at room temperature) is much larger than l, thus spatial superpositions decohere exponentially in time with a characteristic time

$$\tau_{\mathrm{bb}} = \frac{1}{\Lambda_{\mathrm{bb}} l^2}, \tag{6.62}$$

which is sensitive to the superposition size l. The factor Λ_{bb} depends on the material properties of the mass as well as its temperature and that of the environment. If the environment and the mass are at the same temperature T then the factor is

$$\Lambda_{\mathrm{bb}} = \frac{8!8\zeta(9)}{9\pi} cR^6 \left(\frac{k_B T}{\hbar c}\right)^9 \mathrm{Re}\left[\frac{\epsilon-1}{\epsilon+2}\right]^2 + \frac{32\pi^5}{189} cR^3 \left(\frac{k_B T}{\hbar c}\right)^6 \mathrm{Im}\left[\frac{\epsilon-1}{\epsilon+2}\right], \tag{6.63}$$

where ζ denotes the Riemann zeta function and ϵ is the dielectric constant of the nanoparticle's material at the thermal frequency. We take $\epsilon = 5.3$ like that of diamond [24] for the purposes of this estimation. Plugging in the radius of 30 μm of the masses under consideration and the superposition size 10^{-1} μm, we have

$$\tau_{\mathrm{bb}} \approx \frac{2 \times 10^5 \ \mathrm{s}}{(T/\mathrm{K})^9}. \tag{6.64}$$

A coherence time of about 1 s, one order of magnitude above t of table 6.1, will require the temperature to be below 4 K.

Imperfect vacuum

The thermal de Broglie wavelength of a typical gas molecule ($\approx 10^{-10}$m for He at 4K) is many orders of magnitude below the superposition size l considered here, thus a single collision can acquire full which-path information and entail full loss of coherence. The exponential decay rate of the superposition is in this case independent on the size l of the superposition, with a characteristic time

$$\tau_{\mathrm{gas}} = \frac{\sqrt{3}}{16\pi\sqrt{2\pi}} \frac{\sqrt{2m_g k_B T}}{PR^2} \tag{6.65}$$

in a gas at temperature T, pressure P of molecules of mass m_g. Assuming the gas is entirely made of helium, and setting the highest possible value for the temperature according to the previous section, we get

$$\tau_{\mathrm{gas}} \approx \frac{10^{-17} \ \mathrm{s}}{P/\mathrm{Pa}}. \tag{6.66}$$

Thus a coherence time of $10\,t = 1$ s requires a pressure of 10^{-17} Pa. This is a regime of extremely low pressure and may present the most serious challenge for any experiment that involves setting masses of this scale in path superposition. To put things in perspective, pressures of the order 10^{-18} Pa are found in nature in the warm-hot intergalactic medium [182], while the interstellar medium pressure is at the range of 10^{-14} Pa [98]. On the other hand, pressures as low as 10^{-15} Pa at 4 K have been reported since the 1990's in experiments employing cooling magnetic traps [109, 110]. In a similar context to ours, the contemporary GME detection proposals quoted above require pressures of 10^{-15} Pa at 0.15 K [34]. Finally, the cryogenic requirements found in this section can be relaxed if the path superposition can be achieved faster. From equations (6.53) and (6.54), if a stronger magnetic field can be used this will require shorter coherence times.

6.6 Discussion of the hypothesis

At first sight, the hypothesis

$$\delta\tau = n\, t_{\mathrm{P}} \tag{6.13}$$

mimics the naïve picture of a tiny clock ticking at a constant rate, with a lapse t_{P}. This simple physical picture of the quantum mechanical phase as a sort of intrinsic "clock" ticking at planckian time intervals is appealing in its simplicity and does not depend on any particular model of quantum gravity. Thus, in our opinion, it is on its own right worth being looked at.

Whether this hypothesis is backed by a physical theory of time is unclear. In the well corroborated fundamental paradigms of general relativity and quantum mechanics, time is modelled as a continuous variable. However, in a UV completion for quantum gravity, one can reasonably expect a modification of the notion of time at planckian scale. We discuss two main avenues by which the continuous time can become discrete:

A. Instead of a smooth spacetime, consider it instead an effective description on large scales, that emerges from an underlying discrete lattice.

B. Promote time to a quantum observable with a discrete spectrum.

A. Most straightforwardly, (6.13) can be taken *prima facie* to arise from a kind of classical time discreteness. Assuming that the notion of proper time τ of general relativity becomes discrete in a linear sense, with regular spaced planckian time intervals, then also *differences* of proper time $\delta\tau$ will display a similar behaviour, from which (6.13) follows. This assumption is made for instance in the programme of Digital Physics [276], which advocates that space may be nothing but a grid.

Of course, such a 'classical' discreteness would manifestly break Lorentz invariance. It might be already possible to set upper bounds on the discreteness of time from the limits set on Lorentz invariance violations by the study of the dispersion relations of light [2, 5, 143, 181].

Before discussing possible implications of quantum theory, a comment on the intermediate case of a classical but stochastic spacetime. For instance, if spacetime can be described by a single causal set, stochastic fluctuations of planckian size in proper times are to be expected [88, 216, 246]. Because of the statistical nature of the time measurement proposed here, finding a continuous behaviour for $\delta\tau$ would not necessarily exclude the possibility of a classical discreteness. It could just be masked by stochastic fluctuations.

B. Turning to the quantum theory, the discreteness of time may appear as the discreteness of the spectrum of some time operator. Contrary to general belief, Pauli's argument [196] has not ruled out the possibility of a time-operator but rather stressed the subtlety of its definition [111].

There are two main candidates for being the relevant time observable here: the proper time interval τ in each branch, and the difference of proper time $\delta\tau$ between the branches. Then in both cases the question of which spectrum is to be expected should be answered.

Equation (6.13) can be regarded as the assumption of the linearity of the spectrum. For comparison, this is very different from the energy spectrum of the hydrogen atom $E_n \propto -1/n^2$ but it is very similar to that of the harmonic oscillator $E_n \propto n$. If the spectrum of τ is linear, then so is the spectrum of $\delta\tau$, which is what we assumed in the main analysis with equation (6.13). It does not matter, in this case, whether it is τ or $\delta\tau$ which is taken as the relevant quantum observable. On the contrary, for a non-linear spectrum, this question is crucial. As said earlier, the assumption of linearity is natural in the sense that it mimics the ticking of a clock, but it is not really backed so far by any theory of quantum gravity.

In Loop Quantum Gravity (LQG) the spectrum of the length, area and volume operators are famously discrete [229]. Discreteness of time may arise in a similar fashion from this theory, although nothing has been proven yet. There is also a debate on whether discreteness in the spectrum of observables survives the implementation of the hamiltonian constraint [87, 224]. The hypothesised linear behaviour is similar to the spectrum of the area operator in LQG [230]

$$A_j = 8\pi\gamma l_{\mathrm{P}}^2 \sqrt{j(j+1)}, \quad j \in \mathbb{N}/2, \tag{6.67}$$

where γ is a fundamental constant called the Immirzi parameter. There are indications that length has a spectrum that goes as a square root progression in j [26]. Geometrically, we would expect time to behave similarly to a length. In such a case, it will make all the difference whether the square-root behaviour applies to the proper time itself

$$\tau = \sqrt{n}\, t_{\mathrm{P}}, \tag{6.68}$$

or the difference of proper time

$$\delta\tau = \sqrt{n}\, t_{\mathrm{P}}. \tag{6.69}$$

We first analyse the consequences of equation (6.68) on the visibility of the plateaux. We work in planck units and take $l \ll d$ according to the parameters of table 6.1, although the same result can be obtained without this assumption. The proper times τ_{far} and τ_{close} of the branch in which M and m are a distance $d+l$ and d apart are given in terms of laboratory time according to general relativity by

$$\tau_{\mathrm{far}} = t\sqrt{1 - \frac{2M}{d+l}} \qquad \tau_{\mathrm{close}} = t\sqrt{1 - \frac{2M}{d}}. \tag{6.70}$$

These are very large compared to the planck time, as we are in the weak field regime and t cannot be smaller than the period of the sharpest atomic clock. Let's now impose the discretisation (6.68)

$$\tau_{\mathrm{far}} = \sqrt{n+k}, \qquad \tau_{\mathrm{close}} = \sqrt{n} \tag{6.71}$$

where

$$n + k = \left\lfloor \left(1 - \frac{2M}{d+l}\right) t^2 \right\rfloor \qquad \text{and} \qquad n = \left\lfloor \left(1 - \frac{2M}{d}\right) t^2 \right\rfloor. \tag{6.72}$$

Equation (6.13) is thus replaced by

$$\delta\tau = \left(\sqrt{n+k} - \sqrt{n}\right) t_{\mathrm{P}}. \tag{6.73}$$

The condition $l \ll d$ implies that $k \ll n$, so that the equation above simplifies to

$$\delta\tau \approx \frac{k}{2\sqrt{n}}. \tag{6.74}$$

So, a square-root behaviour for the spectrum of τ leads to a linear behaviour for $\delta\tau$. Unfortunately, the factor of $\sqrt{n}$ in the denominator means that different values of $\delta\tau$ are exceedingly close to each other, making the experiment impossible in our proposed setup.

We now consider the case (6.69). We have

$$n = \left\lfloor \left(\frac{t}{\beta}\right)^2 \right\rfloor, \tag{6.75}$$

so that

$$P_+^{h'} = \frac{1}{2} + \frac{1}{2}\sin\left(\frac{m}{m_\mathrm{P}}\sqrt{\left\lfloor \left(\frac{t}{\beta}\right)^2 \right\rfloor}\right). \tag{6.76}$$

This behaviour is plotted next to that of the main hypothesis in figure 6.5. For small values of t/β, the plot of $P_+^{h'}$ is the same as the one of P_+^h, studied in the previous sections. For larger values of t/β, both the width of the plateaus and the steps between them are smaller. Thus, the detection of such a discreteness is of similar difficulty so long as $t/\beta < 10$.

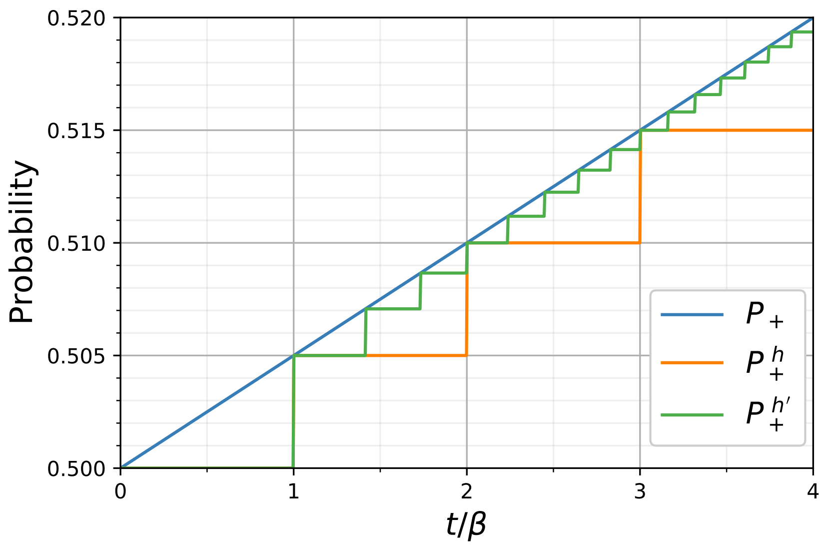


Figure 6.5. Plot of P_+ as a function of t/β with an alternative hypothesis. We take $m = 10^{-2} m_\mathrm{P}$. Blue curve: $\delta\tau$ takes continuous values. Orange curve: $\delta\tau = n\, t_\mathrm{P}$ as considered in the main text. Green curve: $\delta\tau = \sqrt{n}\, t_\mathrm{P}$, as motivated from LQG in this section.

6.7 Conclusion

In this chapter, we have devised an experiment that would probe a hypothetical granularity of time at the planck scale. We have also carried out a feasibility analysis. First, we have determined a set of constraints that would ensure the visibility of the plateaux in the plot of the probability $p_+(t/\beta)$. These constraints are expressed as a set of inequalities on the experimental parameters. Second, based on current claims in the experimental physics literature, we have shown that there exists a reasonable range of parameters that satisfy the constraints. The obtained values are gathered in table 6.1. Finally, we have determined the temperature and pressure conditions required to avoid too fast decoherence.

Perhaps surprisingly, conclude that the proposed experiment is a feasible task for the foreseeable future. In particular, it is of comparable difficulty to contemporary experimental proposals for testing the non-classicality of the gravitational field. Nevertheless it remains difficult, and will require pooling expertise in adjacent experimental fields.

The possibility of probing planckian time without involving extremely high energies may be a disturbing idea to many physicists. However, the history of physics shows examples where scientists have gained knowledge at a physical scale that was widely believed to be unreachable with the available technology at the time. The first example is when Einstein proposes a way to measure the size of atoms by observing the brownian motion of mesoscopic pollen grains [94]. Another example is when Millikan shows that the electric charge comes in discrete packets, and measures the charge of the smallest packet (the electron) [174, 175]. Again, such a feat was realised through the observation of the mesoscopic motion of charged drops of oil. In both cases, as in our proposal, the scale of discreteness was reached through mesoscopic observables thanks to two leverage effects: an algebraic game involving very small or very big constants and a statistical game involving the collection of many events.

The importance of realising the proposed experiment lies primarily in the ground-breaking implications of potentially discovering a granularity of time at the planck scale. A negative result would also have significant implications, guiding fundamental theory. Finally, an easier version of the experiment with relaxed constraints would remain of profound interest, setting new bounds on the continuous behaviour of time.

Chapter 7

The arrow of time in operational formulations of quantum mechanics

Classical mechanics is invariant under time reversal: its elementary laws do not distinguish past from future. The observed arrow of time is a macroscopic phenomenon that depends on the use of macroscopic variables and the contingent fact that the entropy defined by these variables was lower in the past. Is the same true for quantum mechanics? On the one hand, the Schrödinger equation is time-reversal invariant and so is quantum field theory (up to parity transformation and charge conjugation). Elementary physics is time reversal invariant and the source of time orientation is again macroscopic and entropic. Elementary quantum phenomena do not carry a preferred arrow of time. On the other hand, as remarked in section 2.4.3, the formalism of quantum theory is often defined in a markedly time oriented way.

In this chapter, we address this tension between the physics and the formalism. We investigate the reason for the time orientation of the quantum formalism and show that the tension can be resolved. The asymmetry in the formalism is due to the inherent directionality in the process of inference, which is only indirectly related to the arrow of time.

Let us start by noting that in any inferential problem there is an asymmetry between what is known (the data) and what is unknown (the desiderata). Let us call this directionality the *arrow of inference*. The arrow of inference is not necessarily aligned with the entropic arrow of time. The arrow of inference may be pointed towards the past as well as towards the future. Quantum phenomena are such that we can only compute conditional probabilities, so quantum theory inherits the asymmetry between data and desiderata.

Quantum theory allows us to compute the probability of future events from past ones, but it also allows us to compute the probability of past events from future ones. As we illustrate in detail below, quantum theory does not distinguish between these two tasks. In contrast, the users of quantum theory are generally more interested in predicting the future than postdicting the past. This is because we live in thermodynamically oriented world that has abundant macroscopic traces of the past but not of the future [211, 228]. And that is why, in most problems, the arrow of inference points in the same direction as the arrow of time. It is no surprise that we have designed formulations of quantum theory that conflate the two arrows. Nevertheless, ignoring the distinction can be a source of confusion.

We will focus on formalisms used in quantum information [68, 73, 183]. These are designed to study information processing tasks and the correlations that agents can achieve by sharing and manipulating quantum systems. As mentioned in chapter 2, this approach has lead to a wealth of insights, both of theoretical and technological value [16, 17, 21, 22, 32, 34, 56, 86, 96, 116, 122, 134, 164, 242, 267]. Of particular interest are the information-theoretic reconstructions of quantum theory [57, 74, 124, 125, 132, 133, 146, 171, 187, 239] mentioned in chapter 3. These derive the formal Hilbert space structure of quantum theory from simple physical principles. With the notable exception of Ding Jia's [146], the reconstructions either start by considering a space of theories that is intrinsically time-oriented [74, 124, 125, 132, 133, 171] or introduce the time orientation explicitly as a postulate [57, 187, 239] ("no signalling from the future"). There is nothing wrong with time-oriented formalisms designed[1] to study time-oriented questions. However, the reconstruction effort is often motivated by saying that a reconstruction can offer natural ways to look for a generalisation of quantum theory. For example, it might aid in formulating a theory of quantum gravity, in which the existence of a background spacetime cannot be taken for granted [82]. While we do not know if quantum gravity will be a time-reversal symmetric theory, it seems unwise to impose time-reversal asymmetry on the onset, especially if motivated by laboratory physics, and not elementary processes.

The argument of this chapter proceeds as follows. We start with the uncontroversial assumption that the Born rule yields *prediction* probabilities: conditional probabilities for future events, given past ones. We apply standard probability theory to find formulas for *postdiction* probabilities: probabilities about the past, given the future. We do not need to postulate these probabilities: we derive them from the prediction probabilities. In section 7.2, we show that for closed quantum systems the probabilities for prediction and postdiction are identical, a property we call *inference symmetry*. The Born rule can be used in both directions of time without modification, contrary to what is sometimes stated. In section 7.3, we discuss open quantum systems, where this symmetry is hidden. In that case, the prediction and postdiction probabilities differ. However, the difference is dictated by the asymmetries in the *inferential* problem, not by the arrow of time, once again contrarily to what is often stated in the literature. Unitary quantum mechanics is both time-symmetric and inference-symmetric. In section 7.4, we investigate the same question for *quantum channels*, the more general evolutions featuring in operational formulations of quantum theory used in quantum information. Quantum channels are not, in general, inference-symmetric. Yet, by shifting the Heisenberg cut to include part of the apparatus, we show that the inference-asymmetry of quantum channels stems from asymmetries in the inferential data. In section 7.5, we relate the tasks of postdiction with passive and active time-reversals, and discover

[1] During the development of this work, Schmid, Selby, and Spekkens [235], and Hardy [126] have proposed formalisms in which the physical and the inferential aspects of a theory can be separated and make space for time-symmetric physics. The arguments presented in this chapter can be seen as an additional motivation for these new frameworks.

that quantum channels can also be seen as shorthands for calculations about the past. We combine all insights in section 7.6 to show that the asymmetries of the quantum information formulations do not stem from an arrow of time intrinsic to all quantum systems, but from the asymmetry inherent in the process of inference. The time-asymmetry of the operational formalisms used in quantum information theory is that of the time-oriented macroscopic agents that set up the experiments. At the end of the chapter, we briefly discuss the time orientation of other formulations of quantum mechanics.

In appendix A, we offer a brief review of the history of the subject of time-reversal invariance in quantum theory; this allows to situate this chapter in a broader context. This chapter is based on [83].

7.1 Prediction and postdiction

Quantum indeterminism is time-reversal invariant. In presenting the probabilistic nature of quantum theory, we often emphasise that the future of a quantum system is not entirely determined by its past. It certainly is true that the outcomes of future interactions with a quantum system are uncertain, given the details of past interactions. What is less often recognised is that the converse is equally true: given the details of present interactions, the past ones are uncertain. This was already pointed out a long time ago [95]: the irreducible indeterminism of quantum phenomena cuts both ways, leaving both the past and the future uncertain, given data about the present. This has well-known practical consequences, such as the impossibility of deterministic state-discrimination and the no-cloning theorem [183]: interactions with a quantum system do not allow us to guess with certainty how it was prepared.

As we shall see below, the past of a quantum system is *quantitatively* as uncertain as its future: the probabilities calculated using the Born rule can be applied to both predict and postdict. Quantum theory, in fact, does not distinguish *a priori* the tasks of prediction and postdiction and we might say that there is a fundamental "unpostdictability" of the behaviour of quanta.

Let us operationally define what we mean by prediction and postdiction using two related tasks. In both tasks, a friend prepares a quantum system in an initial configuration, allows it to undergo a given transformation, measures it, and finds it in some final configuration. The friend then gives us some information about these events, and asks us to guess the rest. In the first task, we are asked to guess the outcome of the measurement, given the initial configuration and the details of the transformation and of the measurement. In the second task, we guess the outcome of the preparation, given the outcome of the measurement, the details of the transformation, and the set of possible initial states.

Definition 1 (Prediction task) *Given a preparation, a map, a test, and the outcome of the preparation, compute the probabilities for the outcomes of the test.*

Definition 2 (Postdiction task) *Given a preparation, a map, a test, and the outcome of the test, compute the probabilities for the outcomes of the preparation.*

These are inferential[2] tasks, in which we use the available information to make educated guesses. The two tasks share the same physical process, the only difference between the two is inferential.

While the setting of these tasks might appear artificial at first, a moment of thought reveals that it serves as a useful shorthand for physically relevant situations. In fact, postdiction has been extensively studied before [11, 12, 101, 198] and has a number of practical applications; see [219, 247, 268] and references therein.

In the following, we study the relation between these two tasks, as it captures the role played by the arrow of time in quantum theory. First, we consider the case of closed systems, namely when the system under consideration is isolated between preparation and observation. Then we consider open systems, namely when we ignore some degrees of freedom, such as environmental degrees of freedom. Finally, we extend the analysis to the more general case in which the notions of preparation, evolution and measurement are subsumed in the more general idea of *operation* used in quantum information and quantum foundations.

7.2 Closed systems

In this section, we assume that the friend prepares the system by determining the values of a maximal set of compatible observables and does the same at observation. We also assume that the system under consideration is isolated between preparation and observation. Therefore, the preparation and test are represented by orthonormal bases of the Hilbert space associated with the quantum system and the transformation is represented by a unitary transformation. We will relax these assumptions in following sections.

In this case, the Born rule is equally good for predicting the future given the past and postdicting the past given the future [259]. Since this fact is not universally known, we derive it assuming only the uncontroversial fact that the Born rule can be used to predict the future.

We denote by $\{a_i\}_{i=1}^d$ and $\{x_i\}_{i=1}^d$ the bases of the preparation and test, respectively (although we drop the basis indices when they are not strictly needed, to keep the notation cleaner). The solution to the prediction task with the unitary evolution U, and the outcome of the preparation a is given by the Born rule: the probability for the outcome x of the test is

$$P_{pre}(x|a,U) = |\langle x|U|a\rangle|^2. \tag{7.1}$$

The solution to the postdiction task is obtained from the solution of the prediction game and standard probability theory. By Bayes' theorem

$$P_{post}(a|x,U) = \frac{P_{pre}(x|a,U)P(a)}{P(x)}, \tag{7.2}$$

[2]We adopt a broadly bayesian understanding of probability in this chapter. Probabilities represent beliefs about the situation at hand, given the data; quantum-mechanical transition probabilities are thought as the best guide for inference. The latter belief is justified by the theory's success. If we want to think of probabilities in frequentist terms instead, we can imagine the friend repeating the setup many times, ensuring a uniform distribution in the initial or final configuration and interpret the calculated probabilities as relative frequencies of outcomes in an ensemble of trials, in the limit of infinite trials.

where $P(a)$ is the prior probability on the initial configuration and $P(x)$ is the probability of the final configuration given the prior. Since all we know in the postdiction task is the basis of the preparation, we have

$$P(a_i) = \frac{1}{d} \tag{7.3}$$

for all $i = 1, \ldots, d$. The *a priori* probability of the outcome of the test is computed summing over all possible initial states:

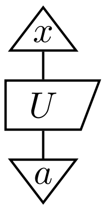
$$P(x) = \sum_{i=1}^{d} \frac{1}{d} P_{pre}(x|a_i, U) = \frac{1}{d}, \tag{7.4}$$

where we used the fact that the evolution is unitary. The postdiction probability is then computed from (7.2):

$$P_{post}(a|x, U) = |\langle x|U|a\rangle|^2 = P_{pre}(x|a, U), \tag{7.5}$$

pictorially,

$$P_{pre}(x|a, U) =$$



$$= P_{post}(a|x, U). \tag{7.6}$$

Thus for a closed quantum system, the solution to the prediction and postdiction tasks is given by the same formula.

Note that the flat prior (7.3) is crucial in the derivation above; had the prior been different, the postdiction probabilities would be different from the prediction probabilities. However, assuming a different prior would introduce an inappropriate asymmetry between the two tasks. Our objective is to treat the prediction and postdiction tasks on equal footing. When we predict the result of a measurement using the Born rule, we do not assume any prior knowledge on the result of the experiment besides the space of alternatives. Therefore, in a postdiction task we do not assume any prior knowledge of the result of the preparation besides the space of alternatives, hence the flat prior. The flat prior does not imply that the input system was prepared in the maximally mixed state, it is simply the probability distribution that represents the prior knowledge in the postdiction task.

For a system evolving unitarily between the preparation and the observation events, later events are uncertain given the earlier event *and* earlier events are uncertain given later events, and the probabilities are given by the same formula. The Born rule does not distinguish the past from the future: it allows to calculate the probability of an event given another event, no matter their order in time.

Let us formalise this property:

Definition 3 (Inference symmetry) *A transformation Φ is inference symmetric if, for any two orthonormal bases $\{a_i\}_{i=1}^{d_A}$ and $\{x_i\}_{i=1}^{d_X}$ for the input and output spaces respectively, the prediction and postdiction probabilities are identical:*

$$P_{pre}(x_i|a_j, \Phi) = P_{post}(a_j|x_i, \Phi) \tag{7.7}$$

Quantum unitary evolution is inference-symmetric. Why is this time-symmetric aspect of unitary evolution rarely emphasised?

In most practical situations, we don't need to use the flat prior when guessing the past. We can do better, because there are macroscopic traces of the past, like our memories or entries in a notebook. Additionally, in most laboratory experiments, the initial distribution is known or chosen so that frequency of events in the ensemble is far from uniform.

The fact that macroscopic traces are records of the past and not the future and the fact that an experimenter's choice can affect the future and not the past are both macroscopic phenomena that pertain to the irreversible physics of the macroscopic world surrounding the experiment [227, 228], not to the quantum dynamics, which by itself does not know the arrow of time.

Some authors go so far as to say that the Born rule does not work 'backward in time' and see it as a fundamental asymmetry in the theory [192] and that quantum theory needs to be modified, or extended, to make it symmetric. But this is too quick. If we do not assume any knowledge or bias in the past, (7.5) is indeed the correct formula to use according to quantum theory. In turn, the validity of the Born rule in predicting the future relies on the same assumptions about the future. Namely, if we did have some knowledge of the future, then (7.1) would not be the best formula to make predictions. For example, if the detector does not detect certain states, the Born rule fails.

The presence of records of the past is not a property of quantum theory *per se*, or of the behaviour of a single quantum, but a property of what surrounds the quantum. See also [4, 259] for early examples of this argument.

7.3 Open systems

Let us now consider the case when the system we deal with is not isolated. An open quantum system can always be seen as a part of a larger closed quantum system. To study this case, consider a tensor decomposition of the input $A \otimes B$ and output $X \otimes Y$ Hilbert spaces. The tasks we consider now regard computing probabilities restricted to some of these subspaces. Denote by d_A, d_B, d_X and d_Y the dimensions of the respective spaces and with $\{a_i\}$, $\{b_i\}$, $\{x_i\}$, and $\{y_i\}$ bases on them. The evolution between input and output space is represented by the unitary U. By the results of the previous subsection, this process is inference-symmetric with the solution

$$P_{pre}(xy|ab, U) = |\langle xy|U|ab\rangle|^2 = P_{post}(ab|xy, U). \tag{7.8}$$

Suppose that we agree with our friend to ignore the subspace Y of the outcome space and compute only the probability of finding x as the outcome of the test on X. This simulates the situation in which our system gets entangled with some other system that is subsequently ignored, like when information leaks into the environment. Note that the difference between a closed and an open system is in the inferential data, not in the physical system. We can solve the prediction task by computing the marginal probability

$$P_{pre}(x|ab,U) = \sum_{i=1}^{d_Y} P_{pre}(xy_i|ab,U), \tag{7.9}$$

where we sum over the space Y we decided to neglect. Similarly we solve the postdiction task with the same unitary evolution, with only knowledge on the outcome of the test on the space X, weighting the original postdiction probabilities with a flat prior:

$$P_{post}(ab|x,U) = \sum_{i=1}^{d_Y} \frac{1}{d_Y} P_{post}(ab|xy_i,U). \tag{7.10}$$

We can use the inference symmetry of the closed system (7.8) to relate the two expressions above:

$$P_{post}(ab|x,U) = \frac{1}{d_Y} P_{pre}(x|ab,U). \tag{7.11}$$

Thus the prediction and postdiction probabilities are no longer equal once part of the output system is ignored.

Suppose now that we agree with our friend to neglect the B part of the input space. This corresponds to the situation in which a system in an unknown state interacts with our original system. Again, the difference is in the inferential data, not the physical setup. The prediction probabilities are obtained by assigning a flat prior to the system B:

$$P_{pre}(xy|a,U) = \sum_{i=1}^{d_B} \frac{1}{d_B} P_{pre}(xy|ab_i,U), \tag{7.12}$$

while the postdiction probabilities are

$$P_{post}(a|xy,U) = \sum_{i=1}^{d_B} P_{post}(ab_i|xy,U). \tag{7.13}$$

The two are again related using (7.8):

$$P_{post}(a|xy,U) = d_B P_{pre}(xy|a,U). \tag{7.14}$$

Again, the two probabilities are different.

We can similarly analyse the case where we agree with our friend to neglect the result of the preparation in B and the outcome of the test on Y, which simulates a situation in which the system is open to influences from an unobserved quantum system. The prediction and postdiction probabilities again differ by a simple normalisation constant:

$$d_Y P_{post}(a|x,U) = d_B P_{pre}(x|a,U). \tag{7.15}$$

Note that the probabilities are equal only when $d_B = d_Y$, so that $A \equiv X$. When the input and output spaces are treated symmetrically, prediction and postdiction tasks are symmetric.

Crucially, the normalisation factor that makes the two kinds of probabilities different does not depend on time: if we neglect a subsystem B at time t_1 and another system Y at time t_2, then the probabilities for the values at t_2 given the values at t_1 are d_Y/d_B times the probabilities of guessing the values at t_1 given the values at t_2. This has nothing to do with a pre-established direction of time. Indeed, it is true regardless of whether $t_1 < t_2$ (as in the example above) or $t_1 > t_2$.

In the general case, the asymmetry between the prediction and postdiction tasks arises because of an asymmetry in the inferential data, not because of an intrinsic asymmetry in the physics or the evolution of the system. Indeed, in all these cases, the underlying process U is inference-symmetric. The inferential tasks are asymmetric only when the inferential data are asymmetric.

The normalisation of the identity is determined by the arrow of inference

We can rephrase all the probability calculations above in terms of density operators. Under unitary evolution a density operator transforms as $\rho \mapsto U[\rho] := U\rho U^\dagger$. A pure state ψ can be represented as a density operator by the projector $|\psi\rangle\langle\psi|$ and the Born rule can be recast as a trace

$$|\langle xy|U|ab\rangle|^2 = \mathrm{tr}\left(|xy\rangle\langle xy|\;\; U[|ab\rangle\langle ab|]\right). \tag{7.16}$$

Let us start with prediction, the more familiar task. In this language the prediction probability (7.9) can be rewritten as

$$\begin{aligned} P_{pre}(x|ab,U) &= \sum_{i=1}^{d_Y} P_{pre}(xy_i|ab,U) \\ &= \sum_{i=1}^{d_Y} \mathrm{tr}\left(|xy_i\rangle\langle xy_i|\;\; U[|ab\rangle\langle ab|]\right) \\ P_{pre}(x|ab,U) &= \mathrm{tr}\left((|x\rangle\langle x| \otimes \mathbb{I}_Y)\, U[|ab\rangle\langle ab|]\right). \end{aligned} \tag{7.17}$$

The decision to ignore part of the output system is represented by the insertion of the identity operator $\mathbb{I}_Y$, which in this role is called the discard operator of the subsystem Y. This is the classic technique of 'tracing out' a subsystem to ignore its future. Equation (7.12) can be similarly recast as

$$P_{pre}(xy|a,U) = \mathrm{tr}\left(|xy\rangle\langle xy|\;\; U\left[|a\rangle\langle a| \otimes \frac{1}{d_B}\mathbb{I}_B\right]\right), \tag{7.18}$$

where the flat prior is represented by the maximally mixed state, the density operator $\mathbb{I}_B/d_B$. This is also the well known result of doing prediction based on partial information. Note that the two formulas above make it clear that the choice of basis of the ignored system is irrelevant to the computed probabilities.

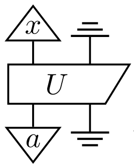
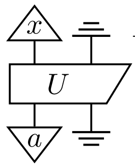
Let us now look at the postdiction probabilities. Using the two equations above together with (7.11) and (7.14), we can immediately write

$$P_{post}(ab|x,U) = \mathrm{tr}\left(\left(|x\rangle\langle x| \otimes \frac{1}{d_Y}\mathbb{I}_Y\right) U[|ab\rangle\langle ab|]\right), \tag{7.19}$$

$$P_{post}(a|xy,U) = \mathrm{tr}\left(|xy\rangle\langle xy|\;\; U[|a\rangle\langle a| \otimes \mathbb{I}_B]\right). \tag{7.20}$$

We see that the identity operator appears again in the ignored systems. However, the normalisation is the opposite of the predictive case.

The discard operator and the maximally mixed state are well known, and are normally only applied to the output and input side respectively. But the normalisation of the identity operator does not reflect the direction of time, the past or the future, input or output. It reflects the direction of inference. This is particularly obvious rewriting in the pictorial calculus:

$P_{pre}(x|a,U) =$  $\frac{1}{d_B}$ $\qquad P_{post}(a|x,U) =$  $\frac{1}{d_Y}$ (7.21)

with $\top$ and $\bot$ representing the identity operator as an output and input respectively. We discard in the direction we guess, and we have the maximally mixed state on the side of the data.

Thus, the way the inference-symmetry appears to be broken in open systems reflects an asymmetry on the inferential data that is, *a priori*, independent on the direction of time. An example illustrates why it is natural that the normalisation depends on the direction of inference and is independent on the direction of time. Consider a system evolving with the identity, *i.e.* nothing happens to it. If we are told the state a of the system and we are not asked to guess anything, then all the probabilities are trivially $1 = \operatorname{tr} |a\rangle\langle a|$. Conversely, if we are only told the system is in one state out of a orthonormal basis, then the probability that it is in a given state a is $\frac{1}{d} = \frac{1}{d} \operatorname{tr} |a\rangle\langle a|$. When we don't guess, all the probabilities are 1; when we are told to guess but we have no clue, our only option is to assume a uniform distribution. This is true regardless of whether we are predicting or postdicting

We often use quantum theory to predict, which is why we generally associate the normalisation factor $1/d$ to the identity operator in the input space. In practice, we are normalising our data. If we were postdicting, we would associate the normalisation factor to the identity in the output space. The operator $\mathbb{I}/d$ does not represent a physical fact, but the probability distribution we use to weight the conditional probabilities computed with quantum theory.

The results of these two sections show that the irreducible quantum uncertainty applies equally to both directions of time. Indeed, when dealing with a closed quantum system, the Born rule gives both the prediction and postdiction probabilities directly. When the prediction and postdiction probabilities differ, they do so because of an asymmetry in the inferential data.

7.4 Quantum operations

The transformations considered above might seem limited in scope to researchers in quantum information and quantum foundations. As we have seen in chapters 2 and 3, in these communities the notions of preparation, evolution, and measurement are subsumed by the more general notion of *operation*, which reflects their more elaborate needs: a more coarse-grained description of quantum processes, independent on the underlying dynamics, the capacity of melding classical and quantum information processing, dealing with classical uncertainty and so on. However, since agents and labs are made of atoms and photons, and the interactions between atoms and photons is satisfyingly described by the unitary evolution and pure state approach, the results of the previous section have bearing on quantum operations too.

After a brief survey of the notion of quantum operation, we solve the prediction and postdiction tasks for a general quantum channel and explain their postdiction asymmetry.

7.4.1 Operations

Let us quickly remind ourselves of the notion of operation to fix some We also take the opportunity to note that this notion is time oriented by design and recall how it relates to the more basic notions.

An *operation* $\mathcal{O}^{A\to X}$, also known as an *instrument*, from an input Hilbert space A to an output Hilbert space X is represented is a set $\{O_i\}$ of completely positive (CP) trace non-increasing linear maps (aka *quantum maps*) from the space $\mathcal{L}(A)$ of linear operators on A to $\mathcal{L}(X)$, satisfying the *completeness equation* (aka the *causality condition*):

$$\forall\rho \in \mathcal{L}(A): \quad \sum_i \operatorname{tr} O_i[\rho] = \operatorname{tr}\rho. \tag{7.22}$$

An operation $\mathcal{O}^{A\to X} = \{O_i\}$ also defines a completely positive, trace-preserving (CPTP) map $\rho \mapsto \mathcal{O}[\rho] := \sum_i O_i[\rho]$. When the operation $\mathcal{O}^{A\to X}$ is applied to a system in state ρ, the outcome i happens with probability given by the generalised Born rule

$$P(i|\rho,\mathcal{O}) = \operatorname{tr} O_i[\rho], \tag{7.23}$$

and the state of the system after this outcome is

$$\rho_i = \frac{O_i[\rho]}{\operatorname{tr} O_i[\rho]}. \tag{7.24}$$

We have already remarked that the completeness requirement (7.22) amounts to a statement of the conservation of probabilities. If the outcome i is unknown, then the state of the system is a mixture of the states above, weighed by the relevant probability:

$$\mathcal{O}[\rho] = \sum_i P(i|\rho,\mathcal{O})\rho_i = \sum_i O_i[\rho]. \tag{7.25}$$

If the output space of an operation $\mathcal{O}^{A\to X}$ coincides with the input space of another operation $\mathcal{M}^{X\to Y}$, these two operations can be sequentially composed forming a new operation $(\mathcal{M}\circ\mathcal{O})^{A\to Y} = \{M_j \circ O_i\}$. If the state of the input system is ρ, the probability of the outcome ij is

$$P(ij|\rho,\mathcal{M}\circ\mathcal{O}) = \operatorname{tr} M_j[O_i[\rho]]. \tag{7.26}$$

Operations can also be composed in parallel using the tensor product structure of the underlying Hilbert spaces.

A *preparation* of a system associated with a Hilbert space A is an operation $\mathcal{P}^{I\to A}$. By a simple mathematical isomorphism, quantum maps from I to A can always be associated with positive linear operators on A, so that any preparation can be represented by a subset $\{\rho_i\} \subset \mathcal{L}(A)$ such that $\sum_i \operatorname{tr} \rho_i = 1$. Above, we have considered only preparations of the form $\{|a_i\rangle\langle a_i| / d_A\}_{i=1}^{d_A}$, where $\{a_i\}$ is an orthonormal basis for A.

A *test* on the same system is an operation $\mathcal{T}^{A \to I}$ from the Hilbert space A to the trivial Hilbert space. By the isomorphism above, and the Riesz representation theorem, tests are often also represented by a collection of positive operators $\{\sigma_j\} \subset \mathcal{L}(A)$, such that $\sum_j \sigma_j = \mathbb{I}_A$ with their actions on the state given by $\rho \mapsto \operatorname{tr} \sigma_j \rho$.

An operation $\Phi^{A \to X}$ with a single outcome is also called a *quantum channel*, and is represented by a CPTP map. Quantum channels are also called *deterministic* quantum maps, as they have only one outcome. Above, we considered only unitary quantum channels, of the form $\rho \mapsto U \rho U^\dagger$, but much more general ones are possible.

A preparation with a single outcome is also called a *state*. States are by definition represented as unit-trace positive operators, *i.e.* density matrices. A test with a single outcome is also called a *deterministic effect*. There is only one deterministic effect, represented by the identity operator.

Note that this formalism is time-asymmetric *by construction*. The time asymmetry shows up in two ways, reminiscent of the Copenhagen-type interpretations. First, the outcomes $\{i\}$ depend probabilistically on the state of the system in the past: the probabilities calculated in this setting are invariably *prediction* probabilities. Second, the state of a system at any point in time reflects events in the past, and it is independent of the events in the future. The only data assumed to be available is data about the past.

The spaces of states and effects are not isomorphic. This was identified as the main source of time-asymmetry of operational quantum theory in [192], where it was proposed to enlarge the space of effects by not requiring that operations sum to trace-preserving maps. However, from the perspective of this chapter, we understand this asymmetry between states and effects as being the difference between known and unknown in the process of inference. Preparations represent our assumptions in the inferential problem, while tests represent the different propositions about the unknowns. There is no need to remove the distinction between preparations and tests to make quantum theory time-symmetric, all is needed is to recognise that operational quantum theory is geared for prediction: a situation where the data is in the past of the unknowns.

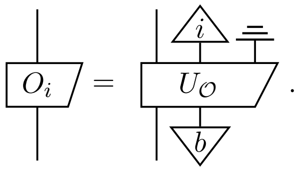
Operational quantum theory is connected to the simpler setting of pure states and unitary evolutions by the concept of *purification*. Any quantum channel $\Phi^{A \to X}$ can be purified [250], meaning it can be represented by a unitary channel $U_\Phi : A \otimes B \to X \otimes Y$ and a pure state b for system B such that

$$\forall \rho \in \mathcal{L}(A): \;\; \Phi[\rho] = \operatorname{tr}_Y U_\Phi[\rho \otimes |b\rangle\langle b|]. \tag{7.27}$$

In other words, any quantum channel can always be understood as a unitary interaction with an ancilla quantum system prepared in a specific way and where part of the output is ignored. In fact, any operation $\mathcal{O}^{A \to X} = \{O_i\}$ can be purified [193], meaning that it is mathematically equivalent to a unitary evolution of the system A in the presence of an ancilla B, in which part of the output system is ignored and part of it is measured on an orthonormal basis. That is, there exists a unitary operator $U_\mathcal{O}$ on the Hilbert space $A \otimes B$ and a decomposition $A \otimes B \equiv X \otimes Y \otimes Z$ and a pure state $|b\rangle\langle b| \in \mathcal{L}(B)$ such that

$$\forall \rho \in \mathcal{L}(A), \forall i : O_i[\rho] = \mathrm{tr}_{YZ} \left((|i\rangle\langle i|_Y \otimes \mathbb{I}_Z) \circ U_\mathcal{O}[\rho \otimes |b\rangle\langle b|] \right), \tag{7.28}$$

where i now labels an orthonormal basis of the Hilbert space Y. This is one of the cases where the pictorial language is definitely clearer:



(7.29)

It is a well-known property of quantum theory that one can always shift the Heisenberg cut to include part of the apparatus in the quantum system under description. This is the physical content of the two mathematical results above. The label i that distinguishes the various outcomes of the operation $\mathcal{O}$ is now seen as labelling the possible values of a *pointer variable*, a part of the apparatus that gets entangled with the system and that, when observed, allows the determination of the state of the system. If one cared enough, one could model every quantum operation explicitly in these terms. But countless purifications are consistent with the same operation, and these low-level details are not important when one is only interested in the effect on the given quantum systems. Quantum operations are useful shorthand.

7.4.2 Prediction and postdiction with quantum channels

Now, let us solve the prediction and postdiction tasks using the generalised Born rule, similarly to how we did for closed quantum systems. Unitary evolution is replaced by a CPTP map Φ. The input and output spaces are A and X respectively, which do not need to be isomorphic. Preparation and measurement are performed on bases $\{a_i\}_{i=1}^{d_A}$ and $\{x_i\}_{i=1}^{d_X}$. The solution of the prediction task is given by the generalised Born rule:

$$P_{pre}(x|a, \Phi) = \mathrm{tr}\, |x\rangle\langle x|\, \Phi[|a\rangle\langle a|]. \tag{7.30}$$

The solution to the postdiction task is found again by Bayesian inversion

$$P_{post}(a|x, \Phi) = \frac{P_{pre}(x|a, \Phi)P(a)}{P(x)}. \tag{7.31}$$

Assigning a flat prior for $P(a)$, we can compute the probability of the data

$$P(x) = \sum_{i=1}^{d_A} \frac{1}{d_A} P_{pre}(x|a_i, \Phi) = \mathrm{tr}\, |x\rangle\langle x|\, \Phi\left[\frac{1}{d_A}\mathbb{I}_A\right], \tag{7.32}$$

which this time is not uniform. Equation (7.31) then becomes

$$P_{post}(a|x, \Phi) = \frac{\mathrm{tr}\, |x\rangle\langle x|\, \Phi[|a\rangle\langle a|]}{\mathrm{tr}\, |x\rangle\langle x|\, \Phi[\mathbb{I}_A]}. \tag{7.33}$$

This formula was also derived in [12]. At first, it looks quite different from the formula for the prediction probabilities. However, it is also a remarkably simple solution. Note that the postdiction probabilities are related to the prediction probabilities by a simple multiplicative factor

$$P_{post}(a|x,\Phi) = f_\Phi(x) \cdot P_{pre}(x|a,\Phi), \tag{7.34}$$

where

$$f_\Phi(x)^{-1} = \sum_{i=1}^{d_A} P_{pre}(x|a_i,\Phi) = \operatorname{tr} |x\rangle\langle x| \, \Phi[\mathbb{I}_A]. \tag{7.35}$$

For a given measurement outcome x, the prediction probabilities are proportional to the postdiction probabilities, up to a fixed normalisation factor $f_\Phi(x)$. Once one has calculated the set of prediction probabilities, one already has the postdiction probabilities, up to this normalisation factor.

Inference symmetry is thus broken in general, and this seems to support the idea there is a fundamental difference between the past and the future in quantum theory. But it is broken in a simple way, reminiscent to the situation in the previous section when we were concerned with partial data in a closed system. Indeed, when we "look under the hood" using purification, we see that this is essentially what is going on. From the purified point of view, there is an asymmetry between inputs and outputs because the purifying ancilla's state is assumed known in input and ignored in output.

7.4.3 Purified task

Let Φ be a quantum channel from Hilbert space A to Hilbert space X. Then there exists a unitary channel $U_\Phi : A \otimes B \to X \otimes Y$ and a pure state b for system B such that

$$\Phi[|a\rangle\langle a|] = \operatorname{tr}_Y U_\Phi[|a\rangle\langle a| \otimes |b\rangle\langle b|]. \tag{7.36}$$

The prediction probabilities for Φ are, by definition, just those of the corresponding pure open system:

$$P_{pre}(x|a,\Phi) = P_{pre}(x|ab, U_\Phi), \tag{7.37}$$

as given in (7.9). However the same is not true for postdiction since, in general,

$$P_{post}(a|x,\Phi) \neq P_{post}(ab|x, U_\Phi). \tag{7.38}$$

This is readily explained by the fact that the initial state of the ancilla is assumed known, or "held fixed." Since we know the ancilla's input state is b, the probability of the data is in fact

$$P(x) = P_{pre}(x|b, U_\Phi). \tag{7.39}$$

Thus, by Bayes' theorem,

$$P_{post}(a|x,\Phi) = \frac{P_{pre}(x|ab,U_\Phi)}{d_A P_{pre}(x|b,U_\Phi)} = \frac{P_{pre}(x|a,\Phi)}{d_A P_{pre}(x|b,U_\Phi)}, \tag{7.40}$$

which indeed immediately translates to (7.33).

We now have a different perspective on the normalisation factor $f_\Phi(x)$: it quantifies the implicit knowledge about the input ancilla system and how this knowledge affects the postdiction task. The specification of the quantum channel Φ contains information about the past of the purifying system system so that postdiction on the quantum channel Φ is equivalent to postdiction on the purified system but with some added information about the input system.

Indeed, we can arrive at the same formula by using the postdiction probabilities for the purified open system and the simple formula $P(a|b) = P(ab)/P(b)$. Indeed one can verify that

$$P_{post}(a|x,\Phi) = \frac{P_{post}(ab|x,U_\Phi)}{P_{post}(b|x,U_\Phi)}, \tag{7.41}$$

by using (7.11) and (7.15).

While we have used an arbitrary purification, the two equations above hold for any purification of the quantum channel Φ. So, whatever the physically appropriate purification might be, the lesson is the same: the inference asymmetry for a quantum channel derives not from an asymmetry of quantum mechanics, but from an asymmetry in the questions asked.

7.4.4 Inference-symmetric channels

We have seen that not every quantum channel is inference-symmetric. Thanks to the solution (7.33) to the postdiction task in the case of a general quantum channel, we immediately see that a channel Φ is inference-symmetric if and only if $f_\Phi(x) = 1$ for all pure states x, namely, if and only if it is identity-preserving:

$$\Phi[\mathbb{I}_A] = \mathbb{I}_X. \tag{7.42}$$

Since quantum channels are trace-preserving, it follows that the input and output spaces are isomorphic. The trace and identity preserving maps are known as the bistochastic quantum maps or unital channels [151, 172], and they are the free operations of the resource theory of quantum thermodynamics [59] with trivial hamiltonians, and the resource theory of purity [251].

Every unitary channel is obviously bistochastic. The noisy operations:

$$\rho \longmapsto \mathrm{tr}_B\, U\left[\rho \otimes \frac{1}{d_B}\mathbb{I}_B\right]. \tag{7.43}$$

form a strict subset of the bistochastic channels [59, 243]. A noisy operation represents the evolution of a system that undergoes unitary interaction with an ancilla about which nothing is known. These channels are exactly those that are simulated by the tasks considered in the previous section when $Y \equiv B$ and both Y and B are left out of the task.

Since the bistochastic channels are the only channels that admit an active time-reversal [55], the equivalence of inference-symmetry and bistochasticity further connects inference-symmetry and time-reversal invariance, as we will see in the next section.

7.4.5 More general preparations

Until now, we limited the preparation to the random element of an orthonormal basis. The reason we considered this is that it reflects the simplest way to interact with a system, namely, to couple to one of its non-degenerate observables. But what if our friend tells us that they prepared one out of n, not necessarily orthogonal, states? We now show how such a situation can also be accounted for using purification. It is possible to skip to the next section without losing continuity with the rest of the chapter.

Say the set of possible initial states states is $\{\psi_i\}_{i=1}^n$, in the d-dimensional Hilbert space A, let U be the unitary evolution and $\{x_j\}_{j=1}^d$ the orthonormal basis of the measurement. Then the prediction probabilities are

$$P_{pre}(x|\psi_i, U) = |\langle x|U|\psi_i\rangle|^2. \tag{7.44}$$

The postdiction probabilities are again found by Bayes' theorem:

$$P_{post}(\psi_i|x, U) = \frac{P_{pre}(x|\psi_i, U)P(\psi_i)}{P(x)}. \tag{7.45}$$

Since we are only told the set of possible states, we assume a flat prior over them:

$$P(\psi_i) = \frac{1}{n}, \tag{7.46}$$

and then the probability for the outcome is

$$P(x) = \sum_{i=1}^n \frac{1}{n}|\langle x|U|\psi_i\rangle|^2 = \operatorname{tr}\left(|x\rangle\langle x|\, U\left[\rho_A\right]\right) \tag{7.47}$$

where we have defined $\rho_A := \sum_i |\psi_i\rangle\langle\psi_i|/n$. And thus

$$P_{post}(\psi_i|x, U) = \frac{P_{pre}(x|\psi_i, U)}{n \operatorname{tr}\left(|x\rangle\langle x|\, U\left[\rho_A\right]\right)}. \tag{7.48}$$

So the prediction and postdiction probabilities are in general different if the space of possible initial configurations does not represent an orthonormal basis.

We can understand this asymmetry again in terms of implicit knowledge. Let B be a n-dimensional Hilbert space, and $\{b_i\}_{i=1}^n$ an orthonormal base for it. Choose an orthonormal basis $\{a_i\}_{i=1}^d$ on A and find a unitary U_P on $A \otimes B$ that maps:

$$U_P : |a_1\rangle \otimes |b_i\rangle \longmapsto |\psi_i\rangle \otimes |b_i\rangle. \tag{7.49}$$

Also define the unitary $U' : A \otimes B \to X \otimes B$ given by

$$U' = (U \otimes \mathbb{I}_B) \circ U_P. \tag{7.50}$$

We can relate the prediction and postdiction probabilities for these two games. For prediction we have:

$$P_{pre}(x|\psi_i, U) = P_{pre}(x|a_1 b_i, U') \tag{7.51}$$

since, for an arbitrary basis $\{y_i\}_{i=1}^n$ of B:

$$\begin{aligned}
P_{pre}(x|a_1b_i,U') &= \sum_{j=1}^{n} \langle xy_j|U'|a_1b_i\rangle \\
&= \sum_{j=1}^{n} |\langle xy_j|U\otimes\mathbb{I}_B|\psi_ib_i\rangle|^2 \\
&= |\langle x|U|\psi_i\rangle|^2\cdot\sum_{j=1}^{n}|\langle y_j|b_i\rangle|^2 \\
P_{pre}(x|a_1b_i,U') &= P_{pre}(x|\psi_i,U),
\end{aligned} \tag{7.52}$$

where we have used the definitions of U' and U_P to obtain the second equality.

For postdiction we have

$$P_{post}(\psi_i|x,U) = \frac{P_{post}(a_1b_i|x,U')}{P_{post}(a_1|x,U')}, \tag{7.53}$$

which is entirely analogous with (7.41). The proof is just a matter of expressing nominator and denominator in terms of probabilities for the original task:

$$\begin{aligned}
P_{post}(a_1b_i|x,U') &= \frac{1}{n}\sum_{j=1}^{n} P_{post}(a_1b_i|xy_j,U') \\
&= \frac{1}{n}\sum_{j=1}^{n} P_{pre}(xy_j|a_1b_i,U') \\
&= \frac{1}{n}P_{pre}(x|a_1b_i,U') \\
P_{post}(a_1b_i|x,U') &= \frac{1}{n}P_{pre}(x|\psi_i,U)
\end{aligned} \tag{7.54}$$

and so

$$\begin{aligned}
P_{post}(a_1|x,U') &= \sum_{i=1}^{n} P_{post}(a_1b_i|x,U') \\
&= \sum_{i=1}^{n}\frac{1}{n}P_{pre}(x|\psi_i,U) \\
P_{post}(a_1|x,U') &= \mathrm{tr}\left(|x\rangle\langle x|\,U[\rho_A]\right).
\end{aligned} \tag{7.55}$$

Thus, cases where we have more general preparations can be understood in terms of preparations on orthonormal bases. From (7.53), in analogy with (7.41), that the preparation of non-orthonormal states contains some implicit information about an ancilla system that allows to prepare the non-orthogonal states. Again, we see that in this purified game, the only way to achieve postdiction probability 1 is to marginalise over b_i, *i.e.* not guess at all.

This method can be further generalised to the preparation of an arbitrary set of density operators by adding a purifying system that gets traced over in the prediction task, and is assigned a flat prior in the postdiction task. The asymmetry again can be understood as an asymmetry in the assumed knowledge in the tasks.

7.5 Relation between time-reversal and postdiction

We have explored the symmetry between guessing the future and guessing the past. For a closed system, the past is quantitatively as uncertain as the future, in the sense that the Born rule can also be used, without modification, to guess a past event based on information about a future event. In the case of a general quantum channel, this symmetry is broken: the prediction and postdiction probabilities are given by different formulas. However, the reason for this is that the implementation of a general quantum channel requires the preparation of an ancilla system in a given state, so that knowing a channel was implemented confers information about the input system, breaking the symmetry between prediction and postdiction. Quantum probabilities by themselves have no regard for the direction of time.

In this section, we examine another way in which the quantum probabilities do not distinguish between past and future by examining the more familiar notion of time symmetry: that of time-reversal symmetry. There are two notions of time-reversal, which we could call active and passive, or operational and descriptive. In the passive time-reversal of a system, one simply examines the changes in the system in the reverse order, starting from the future and moving towards the past. Applying passive time-reversal to the inference tasks we have been considering amounts to switching from the prediction task to the postdiction task and *vice versa*. The previous sections have thus been an investigation of passive time-reversal symmetry; we proved that passive time-reversal symmetry is equivalent to inference-symmetry. Active time-reversal consists instead in finding a process that undoes the change that was brought by a previous transformation. In the context of the inference tasks, active time-reversal consists in considering a new, time-reversed task.

7.5.1 Time-reversed task with unitary channel

Unitary maps are invertible, and thus for every evolution U of a closed quantum system, there exists a time-reversed evolution given by the adjoint $U^\dagger$.

Definition 4 (Time-reversed task) *Consider a task in which a closed system is prepared in a basis $\{a_i\}_{i=1}^d$ and measured in a basis $\{x_i\}_{i=1}^d$ after undergoing the unitary evolution U. In the time-reversed task, the system is prepared in $\{x_i\}_{i=1}^d$ and measured in $\{a_i\}_{i=1}^d$ after undergoing the evolution $U^\dagger$.*

It follows immediately from the properties of the inner product that

$$P_{pre}(a|x, U^\dagger) = P_{pre}(x|a, U). \tag{7.56}$$

While this is result is trivial to derive, it is nevertheless profound, as it compounds with the inference symmetry

$$P_{pre}(x|a, U) = P_{post}(a|x, U), \tag{7.57}$$

to show how little regard the probabilities of closed quantum systems have about the direction of time. Indeed, given two pure states a and x of the corresponding Hilbert spaces, the *same* quantity $|\langle x|U|a\rangle|^2$ is the solution for four conceptually distinct tasks:

- $P_{pre}(x|a,U)$, for the prediction task with unitary evolution U,
- $P_{post}(a|x,U)$, for the postdiction task with unitary evolution U,
- $P_{pre}(a|x,U^\dagger)$, for the prediction task with unitary evolution $U^\dagger$, and
- $P_{post}(x|a,U^\dagger)$, for the postdiction task with unitary evolution $U^\dagger$.

The familiar Born rule, which is normally thought to apply only to the first case, actually applies to all four of these cases. In each one of them, x can be either the future or the past event, and can either be the known or unknown in the scenario. The situation may be represented by the following diagram:

$$
\begin{tikzcd}
P_{pre}(x|a,U) \arrow[rr, leftrightarrow, "\text{active}"] \arrow[dd, leftrightarrow, "\text{passive}"'] & & P_{pre}(a|x,U^\dagger) \arrow[dd, leftrightarrow, "\text{passive}"] \\
& |\langle x|U|a\rangle|^2 & \\
P_{post}(a|x,U) \arrow[rr, leftrightarrow, "\text{active}"'] & & P_{post}(x|a,U^\dagger)
\end{tikzcd}
$$

Let us now consider the case of open systems and study the time-reversed version of the tasks examined in section 7.3. If we neglect the outcome of the measurement on B, we can write:

$$
\begin{aligned}
P_{pre}(a|xy,U^\dagger) &= \sum_{i=1}^{d_B} P_{pre}(ab_i|xy,U^\dagger) && (7.58)\\
&= \sum_{i=1}^{d_B} P_{pre}(xy|ab_i,U) && (7.59)\\
P_{pre}(a|xy,U^\dagger) &= P_{post}(a|xy,U), && (7.60)
\end{aligned}
$$

where we have used the time-reversal symmetry (7.56) in the second equality, and (7.11) in the third. This equation relates the probability of the prediction task to the probability of the time-reversed postdiction task. In both cases, we are calculating the probability of event a based on knowledge of event xy. However, in one case xy is to the past of a and in the other case the opposite is true. In a similar fashion, we can derive the following equations:

$$P_{pre}(ab|x,U^\dagger) = P_{post}(ab|x,U), \tag{7.61}$$

$$P_{post}(xy|a,U^\dagger) = P_{pre}(xy|a,U), \tag{7.62}$$

$$P_{post}(x|ab,U^\dagger) = P_{pre}(x|ab,U), \tag{7.63}$$

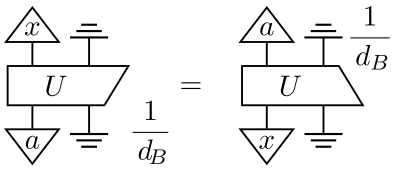
for when only one side is being ignored, as well as

$$P_{pre}(a|x,U^\dagger) = P_{post}(a|x,U), \tag{7.64}$$

$$P_{post}(x|a,U^\dagger) = P_{pre}(x|a,U), \tag{7.65}$$

for when data is being ignored on both sides. Recall the discussion in section 7.3 about the normalisation of the identity that is required to calculate the quantities on the right hand side. The same applies here, which reinforces the lesson: the direction of time is irrelevant to the normalisation of the identity. What matters is the direction of inference:

$P_{pre}(x|a, U) =$



$= P_{post}(x|a, U^\dagger).$ (7.66)

The equation above[3] refers to inference tasks in which we have information about the system A and want to guess something about the system X. It does not matter if A is in the past or the future of X. The answer is the same. There is no way of telling which way the arrow of time is pointing.

7.5.2 Time-reversed task with quantum channel

The natural candidate for the time-reversed task is the one with the adjoint $\Phi^\dagger$ of the quantum channel Φ. In fact, Chiribella *et. al.* proved that this is essentially the unique way of defining the active time-reversal of a quantum channel in a way that reduces to the adjoint when applied to unitaries [55, 58]. We must distinguish two cases, as the adjoint of a CPTP map may or may not be a CPTP map.

The adjoint $\Phi^\dagger$ is trace-preserving if and only if Φ is unit preserving since, by the definition of the adjoint, for all $\rho \in \mathcal{L}(X)$:

$$\operatorname{tr} \Phi^\dagger[\rho] = \operatorname{tr} \mathbb{I}_A \circ \Phi^\dagger[\rho] = \operatorname{tr} \Phi[\mathbb{I}_A] \circ \rho. \tag{7.67}$$

Furthermore, we have seen that the bistochastic channels are exactly the inference symmetric channels. In analogy with the discussion above on the closed systems, for a bistochastic channel Φ, the quantity

$$\operatorname{tr} |x\rangle\langle x| \circ \Phi[|a\rangle\langle a|], \tag{7.68}$$

given by the generalised Born rule, yields the numerical value of four *a priori* conceptually distinct quantities:

- $P_{pre}(x|a, \Phi)$, for the prediction task with bistochastic channel Φ,
- $P_{post}(a|x, \Phi)$, for the postdiction task with bistochastic channel Φ,
- $P_{pre}(a|x, \Phi^\dagger)$, for the prediction task with bistochastic channel $\Phi^\dagger$, and
- $P_{post}(x|a, \Phi^\dagger)$, for the postdiction task with bistochastic channel $\Phi^\dagger$.

Thus postdiction-symmetry is intimately linked with time-reversal invariance even in the context of quantum channels.

In general, however, the adjoint of a CPTP map might fail to be trace non-increasing. So it might not only fail to represent a quantum channel, but may not even be a quantum map, *i.e.* part of an operation. In this case, an active time-reversal of the corresponding quantum channel does not exist, as has been recently shown in [55, 58].

[3] We are using the horizontal reflection of a unitary U to represent its adjoint.

7.5.3 Quantum channels towards the past

A quantum channel might not have an active time-reversed version. Nevertheless, we can learn something striking by looking at the time-reversed task of a *purification* of this quantum channel: CPTP maps can describe *postdictions* in situations in which part of the future data is left implicit and fixed. Consider the time-reversed version of the purified task in section 7.4.3. The system is prepared in some state corresponding to the basis $\{x_i y_j\}$ and measured on some basis $\{a_i b_j\}$ after undergoing the transformation described by $U_\Phi{}^\dagger$. Using the time-symmetry of unitary open systems (7.63), observe that

$$P_{pre}(x|a, \Phi) = P_{pre}(x|ab, U_\Phi) = P_{post}(x|ab, U_\Phi^\dagger). \tag{7.69}$$

That is, the quantity

$$\mathrm{tr}\, |x\rangle\langle x|\, \Phi[|a\rangle\langle a|] \tag{7.70}$$

is the solution to two different tasks:

- $P_{pre}(x|ab, U_\Phi)$ for the prediction task with unitary U_Φ, and
- $P_{post}(x|ab, U_\Phi^\dagger)$ for the time-reversed postdiction task with $U_\Phi^\dagger$.

Thus one can take the quantity (7.70) to relate to an inference towards the past, in a situation in which part of the future events are only implicitly described. This furthers the argument that the inference-asymmetry of the CPTP maps is not an asymmetry related to an intrinsic direction of time in the details of a quantum process, but in an asymmetry in the data about a system. The input of a CPTP map does not necessarily lie to the past of the output, it can also lie to its future.

This last insight will play a major role in dispelling a source of confusion regarding the time-orientation of quantum phenomena.

7.6 The arrow of inference

In this section, we discuss two closely related asymmetric aspects of the operational formalism, encapsulated by two maxims: "there exists a unique deterministic effect" and "no signalling from the future". These expressions reflect mathematical properties of the theory that are often taken as evidence of an asymmetry between the past and the future. Here we show that these properties reflect the asymmetry due to the arrow of inference, which is in principle independent of the arrow of time.

7.6.1 "There exists a unique deterministic effect"

As a mathematical statement, given the definitions, this maxim is correct. A quantum map is deemed *deterministic*, or *causal*, if it is trace preserving. An *effect* is a quantum map from a Hilbert space to the trivial Hilbert space. Thus a *deterministic effect* is an effect that is trace-preserving. It is easy to see that the only such effect is the one that maps a state to its trace, *i.e.* the discard operator. In this precise sense the discard is the only deterministic effect.

As a statement of the irreducible uncertainty of quantum phenomena, the maxim is also correct: the only way to be certain about a prediction in all cases is not to guess at all. In the context of prediction tasks, discarding means not trying to predict anything about the system and is represented by the identity operator as an effect (see section 7.3). Not guessing is the only way to always guess right, and this is the physical content of "there exists a unique deterministic effect."

The uniqueness of the deterministic effect is sometimes understood as the conservation of probability, a mere aspect of inference (see for example [13, 235]). However, the maxim is sometimes taken to signify something fundamental about the distinction between the past and the future [67, 192], and this is not correct. In fact, we can define by analogy what it means to discard something in the past. If discarding in the future means marginalising prediction probabilities, then discarding in the past means marginalising postdiction probabilities. Discarding in the past is also represented by the identity operator when computing probabilities—this time in the *input* side, as we have seen at the end of section 7.3. Indeed, looking at the formula (7.33) for postdiction with a general quantum channel, we see that the only way to have probability 1 is to discard the system, *i.e.* by not guessing at all.

Thus, the uniqueness of the discard operator is not a consequence of the time-orientation of quantum phenomena. It is instead a manifestation of thoroughly time-symmetric quantum indeterminism in the context of prediction.

7.6.2 "No signalling from the future"

This maxim means that the probabilities of the outcomes of a quantum operation do not depend on the nature of a later operation. To lay down some notation, we reproduce the standard proof of this property, which is an immediate consequence of the equations (7.22) and (7.26). In the next section, we comment on the physical reasons for this and show that it too is in fact a property of the arrow of inference, not a property of time.

Suppose a system starts in a state ρ, and the operation $\mathcal{E}^{A\to D} = \{E_x\}$ is applied. The quantity

$$P_{pre}(x|\rho, \mathcal{E}) = \operatorname{tr} E_x[\rho] \tag{7.71}$$

is the probability of observing outcome x.

Suppose instead that the operation $\mathcal{E}$ is immediately followed by another operation $\mathcal{F}^{D\to Z} = \{F_y\}$. The probability for the outcomes x and y is given definition by (7.26):

$$P_{pre}(xy|\rho, \mathcal{F} \circ \mathcal{E}) = \operatorname{tr} F_y \left[E_x[\rho]\right] \tag{7.72}$$

An immediate application of probability theory and the completeness equation (7.22) yields

$$P_{pre}(x|\rho, \mathcal{F} \circ \mathcal{E}) = \sum_y \operatorname{tr} F_y \left[E_x[\rho]\right] = \operatorname{tr} E_x[\rho], \tag{7.73}$$

and we thus have the identity

$$P_{pre}(x|\rho, \mathcal{F} \circ \mathcal{E}) = P_{pre}(x|\rho, \mathcal{E}). \tag{7.74}$$

The probabilities of the outcomes of the first operation are independent on the nature of the second operation.[4] The case in which the operation $\mathcal{F}$ is performed and the case in which it is not are indistinguishable by looking only at what happens at $\mathcal{E}$.

A future experimenter cannot affect the statistics in the present by manipulating a system in the future, hence "no signalling from the future."

What about the opposite direction? By linearity and (7.25) we have

$$P_{pre}(y|\rho, \mathcal{F} \circ \mathcal{E}) = P_{pre}(y|\mathcal{E}[\rho], \mathcal{F}), \tag{7.75}$$

so that the probability of y clearly depends on the first operation $\mathcal{E}$.

Thus the future operation does not affect the probabilities of past events, while the past operation affects the probability of future events. One seems compelled to conclude that this reveals a time-asymmetry in quantum theory. But this conclusion is too quick. In previous sections, we have encountered various situations in which asymmetric aspects of quantum theory should not be ascribed to a difference between past and future, but to the directionality of inference and an asymmetry of the data. We have also seen that the operational formulations assume that the arrow of time and the arrow of inference point the same way. We should be cautious.

7.6.3 "No signalling from the further unknown"

The calculation above in fact shows that the future operation does not affect the *prediction* probabilities of events in its past, while the past operation affects the *prediction* probabilities of events in its future. In other words, if somebody in the past of both operations is trying to guess what would be the outcome of the first operation, they can safely discard any information about the second operation that might be available at that time.[5] Put another way: an event further away from the data does not affect the prediction probabilities closer to the data.

A similar statement holds also when the arrow of inference points toward decreasing time, *i.e.* when doing postdiction. As we have already seen in section 7.5.2, one can understand a CPTP map as a shorthand to calculate a postdiction probability when a future event is implicitly considered fixed. The same is generally true of *any* operation: it can serve as a shorthand to aid the calculation of postdiction probabilities. Then (7.74), understood as a statement about postdiction probabilities, tells us that what happened in the further past does not affect our ability to infer what happens in the closer past. When predicting, an event does not affect prediction probabilities about an event in its past. When postdicting, an event does not affect

[4] Note that the no-signalling from the future property (7.74) can also be seen as a motivation for the definition (7.26) for the probabilities of the outcomes of sequential operations in the first place. Indeed by linearity, these can be rewritten as:

$$\mathrm{tr}\, F_y\,[E_x[\rho]] = \mathrm{tr}\left[F_y\left(\frac{E_x[\rho]}{\mathrm{tr}\, E_x[\rho]}\right)\right] \cdot \mathrm{tr}\, E_x[\rho] \tag{7.76}$$

so that, by setting $\rho_x = E_x[\rho]/\mathrm{tr}\, E_x[\rho]$ one can write:

$$P_{pre}(xy|\rho, \mathcal{F} \circ \mathcal{E}) = P_{pre}(y|\rho_x, \mathcal{F}) \cdot P(x|\rho, \mathcal{E}). \tag{7.77}$$

[5] To be sure, having knowledge of the outcome of the second operation *does* provide an advantage in guessing the outcome of the first one, since

$$P(x|y, \rho, \mathcal{F} \circ \mathcal{E}) = \mathrm{tr}\, F_y[E_x[\rho]]/\mathrm{tr}\, F_y[\mathcal{E}[\rho]]. \tag{7.78}$$

This data however is unavailable to somebody sitting in the past of both operations.

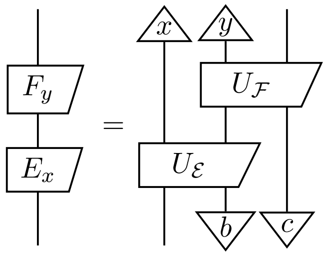
postdiction probabilities of events in its future. In both cases, there is no signalling from the further unknown.

As an illustration, let us consider a purification of the two operations $\mathcal{E}^{A\to D}$ and $\mathcal{F}^{D\to Z}$ above. Let us assume for simplicity that there is no need to discard a part of the output system (by handling the more general case we would reach the same conclusions but with more typographical effort). Then there exists Hilbert spaces B, C, X and Y, unitaries $U_{\mathcal{E}} : A \otimes B \to X \otimes D$ and $U_{\mathcal{F}} : D \otimes C \to Y \otimes Z$, pure states $|b\rangle \in B$ and $|c\rangle \in C$ as well as orthonormal bases for X and Y labelled by x and y respectively, such that for all $\rho \in \mathcal{L}(A)$ and $\sigma \in \mathcal{L}(D)$:

$$E_x[\rho] = \mathrm{tr}_X\, |x\rangle\langle x| \circ U_{\mathcal{E}}[\rho \otimes |b\rangle\langle b|], \tag{7.79}$$

$$F_y[\sigma] = \mathrm{tr}_Y\, |y\rangle\langle y| \circ U_{\mathcal{F}}[\sigma \otimes |c\rangle\langle c|], \tag{7.80}$$

meaning that:



. (7.81)

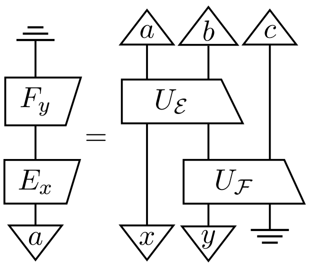
We can write down the prediction probabilities for the purified task in terms of the operations:

$$P_{pre}(xy|abc, U_{\mathcal{F}}U_{\mathcal{E}}) = \mathrm{tr}\, F_y[E_x[a]]. \tag{7.82}$$

Now we can use the property (7.63) of time-reversed unitary tasks to also write

$$P_{post}(xy|abc, U_{\mathcal{E}}^\dagger U_{\mathcal{F}}^\dagger) = \mathrm{tr}\, F_y[E_x[a]]. \tag{7.83}$$

Thus, the quantities $\mathrm{tr}\, F_y[E_x[a]]$ are also postdiction probabilities for a different scenario, in which *future* events b and c are held fixed, and this knowledge is used to guess something happening to their past (x and y):



. (7.84)

In this case, the operation $\mathcal{F}$ contains information about something that happens *earlier* in the system, namely, the interaction $U_{\mathcal{F}}^\dagger$ between the subsystems Y and Z, and the outcome of the measurement of C. All of this information is irrelevant when postdicting only the preparation on X:

$$\sum_y P_{post}(xy|abc, U_{\mathcal{E}}^\dagger U_{\mathcal{F}}^\dagger) = P_{post}(x|abc, U_{\mathcal{E}}^\dagger). \tag{7.85}$$

Thus, when postdicting, the outcome and nature of the earlier operation is irrelevant to the probabilities for the outcome of the later operation.

7.6.4 Why we *can* signal from the past

We have seen that the operation $\mathcal{E}$ affects the prediction probabilities for the outcome of $\mathcal{F}$. How is this related to the notion of *signalling*? Let's imagine two parties, one located where $\mathcal{E}$ takes place and one where $\mathcal{F}$ takes place. Let's call them Eve and Fred. How can Eve send a signal to Fred? The barebones scenario for signalling to the future is that Eve's operation is simply a state preparation of a qubit, and Fred's is a projective measurement. Eve, who knows the basis on which Fred will measure, *chooses* one of the two states so that she fully determines the outcome of Fred's operation. Like this, she can send one bit of information.

Notice, however, that Eve's ability to choose is crucial to this protocol. If she cannot pick what state to send to Fred, she cannot send a message to him. All she can do is try to predict what will be the outcome of Fred's measurement, once—and if—she knows what state she sent him. How does somebody prepare a system in a specific state? In practice, this is done by subjecting the system to a maximal test and discarding the systems yielding unwanted results [202]. Alternatively, one can apply a unitary transformation to the system, conditional on the outcome of the maximal test. Both of these procedures require an increase in the entropy of the universe, as the first involves picking and choosing [173, 227] and the second is an instance of erasure so Landauer's principle applies [152], a rigorous proof of which just appeared [180]. Thus, signalling is a concept beyond single quantum transition probabilities and has its origins in the thermodynamic arrow of time.

We are time oriented creatures, we know more about the past than about the future, we mostly try to guess the future. Quantum probabilities do not care if you are making guesses about the future or the past. They are about predicting what is unknown from what is known. The difference between what is known and what is unknown is at the origin of the time asymmetric maxims of the operational formulations. The first maxim arises from the fact that postdiction scenarios are rarer than prediction ones in practice. If we want to learn about the past, we find there are plenty of records about past events in the present. The existence of traces is not a property of the probabilities of individual quantum systems, so the fact that we rarely have to guess about the past the same way we have to guess about the future is no evidence of a time asymmetry of the physics of quanta.

The grip of the second maxim on the community is more subtle. It rests on the notion of signalling, which is itself tightly linked with ideas of causation and agency, concepts we have strong intuitions for and rely on daily. We humans make choices and these choices influence our future (not our past). The same goes for a lot of systems around us: when my laptop suddenly "decides" to break, it will affect my ability to finish a future paper (not a past one). Is this time-orientation a direct consequence of some time asymmetry in quantum phenomena? Hopefully, by now, the answer is clear. Is causation a fundamental property of the world in some other way? This question has received surprisingly little attention from the physics community at large [211]. This question needs to be addressed carefully if one hopes of extending these operational formalisms to probe physics outside laboratories. The notion of causes always preceding effects is strongly related to notions of agency. And agency is a perspectival property, stemming from a partial description of systems and the presence of an entropy gradient [173, 227]. One should be cautious in extending notions of time oriented causation all the way to elementary physics.

7.7 Time orientation of other formalisms

Before concluding the paper, we add some comments regarding the time orientation in other formalisms.

In the de Broglie-Bohm interpretation [30, 118], the particle and the pilot-wave obey time-reversal invariant dynamics. The Everettian wavefunction [232, 254] also obeys time-reversal invariant dynamics. Branching towards the future is interpreted as a past low entropy condition: systems were uncorrelated in the past, hence the vanishing of von-Neumann relative entropy. Alternatively, it can be interpreted as a perspectival aspect of the Everettian relative state determined by an observation in the present.

The Copenhagen quantum state is correlated with past observations, not future ones, but the state is, of course, unobservable and its ontological status is debated. Its empirical content is given by the probabilities it allows to calculate. If we ask time-symmetric questions, the probabilities we obtain are time-symmetric [4]. The state is assumed to be correlated with past events because we are using past data to infer about the future. If we want to guess about the past, we could just as well use a quantum state correlated with a future event [225, 247, 259].

In QBism [108], quantum theory is interpreted as a means to aid decision making, allowing an agent to calculate the probabilities of the consequences of their interactions with the world. Because decision-making agents play a central role in bringing about the world according to the QBists [106], the ontology of the theory is fundamentally time oriented.

Laboratory measurements generally involve decoherence and amplification of a microscopic phenomenon to the macroscopic realm, both of which rely on the entropy gradient. Views of quantum theory that insist that the only real events are of this kind are therefore time-oriented, even though the probabilities themselves might be time-symmetric. These views also imply that there is no interpretation of the theory outside of the macroscopic approximation.

According to the relational interpretation of quantum mechanics (RQM) [221, 226], facts happen at every interaction between any two systems, but the facts are relative to the systems involved in the interaction. The quantum state only plays a computational role in this interpretation. In the next chapter, we will see how decoherence comes into play to stabilise a fact , so that one might ignore its relational nature, which is manifest in interference effects. Decoherence requires information loss and an increase in entropy. Hence RQM is a time-symmetric formulation of quantum theory, but the dynamics of *relative* facts is time symmetric while the dynamics of *stable* facts is time oriented.

Conclusion

Quantum theory is not about predicting the future, it is about time-symmetric conditional probabilities relating events. The directionality internal to the theory is the arrow of inference, the difference between known and unknown.

There are *formulations* of quantum theory that break time reversal symmetry and use time oriented theoretical notions. These either refer to non-observable entities, or to assumptions about the time orientation of inference problems, or to the entropic time orientation of decoherence. Examples of such formulations are provided by the use of a quantum state determined by *past* interactions in the Copenhagen-type interpretations, and the use of the quantum operations described above. The time orientation of operations is due to them being high-level notions with a built-in assumption about time asymmetric capabilities of the experimentalists.

The time orientation of the formalisms we use is determined by the common boundary conditions we set for the physical processes we study: they come from the assumptions about the agent interacting with the system and the conditions she imposes on it. The agent is not directly modelled in the theory and is instead represented by the inferential boundary conditions and choice of operations, the exogenous variables of [178]. Time orientation is in this way external to the elementary quantum process being modelled. It can, in principle, be entirely accounted for at the level of statistical mechanics as a consequence of the existence of an entropy gradient, namely past low entropy [173, 177, 227, 228].

Some authors [4, 192] have built time-symmetric theories to replace quantum theory. However, the results of sections 7.2 and 7.5 show that quantum theory is already time-symmetric as it is. The transition probabilities calculated with quantum theory are blind to the direction of time. The probabilities of closed quantum systems are inference symmetric and time-reversal invariant. Thus, when accounting for all the relevant degrees of freedom, the predictions of quantum theory are thoroughly time-agnostic. The probabilities of open quantum systems are in general neither inference symmetric nor time-reversal invariant. We have shown that the asymmetry between prediction and postdiction in this case is only a consequence of treating the two problems asymmetrically, by assuming more knowledge in one case than in the other. Both inference asymmetry and the failure of time-reversal invariance of quantum channels can be understood in the same terms. This asymmetry[6] is not intrinsic to the mechanical theory, but is, rather, an asymmetry of the questions we humans ask using the theory.

[6]This situation is reminiscent of the one in classical statistical mechanics, in which one can prove that entropy increases *both* towards the past *and* towards the future of the given initial state [178, 211]. In both situations we use time-symmetrical mechanical laws to extrapolate from a state of limited knowledge to a state at other times. This results in even more limited knowledge. The fact that entropy was lower in the past, is a fact external to the laws of mechanics. And the fact that we are mostly interested in extrapolating towards the future is in turn a consequence of the low entropy in the past.

Part IV

Facts and Objectivity in QM

Chapter 8

Stable facts, relative Facts

8.1 Facts in quantum theory

The common textbook presentation of quantum theory assumes the existence of a classical world. A measurement involves an interaction between the classical world and a quantum system and produces a definite result, for instance a dot on a screen. The result is a fact by itself, but also establishes a fact about a quantum system. For instance, a certain measurement resulting in a definite record establishes that at some time the z-component of the spin of an electron is $L_z = \hbar/2$, which is then a fact about the electron.

Quantum probabilities are probabilities for facts, given other facts. Facts are therefore the arguments of which the probability amplitudes are function. In particular, facts are used as conditionals for computing probabilities of other facts. For instance, if the spin of the electron mentioned above is immediately measured in a direction at an angle θ from the z-axis, the probability to find the value $L_\theta = \hbar/2$ (a fact), given the fact that $L_z = \hbar/2$, is

$$P\left(L_\theta = \frac{\hbar}{2} \,\middle|\, L_z = \frac{\hbar}{2}\right) = \cos^2\left(\frac{\theta}{2}\right). \tag{8.1}$$

In this sense, quantum mechanics is about conditional probabilities relating facts about systems.

Facts ascertained in a conventional measurement are *stable* in the following sense. If we know that one of N mutually exclusive facts a_i $(i = 1 \ldots N)$ has happened, the probability $P(b)$ for another fact b to happen is given by

$$P(b) = \sum_{i=1}^{N} P(b|a_i)P(a_i), \tag{8.2}$$

where $P(a_i)$ is the probability that a_i has happened and $P(b|a_i)$ is the probability for b given a_i. We take equation (8.2) as a characterisation of *stable* facts.

As we saw in section 3.3 textbook presentation of quantum mechanics is incomplete because it assumes the existence of a classical world. An exactly classical world can exist only if current quantum theory has limited validity—for instance if physical collapse mechanisms exist [115, 200], or for some other, still unknown reason. Quantum theory has however been universally successful so far, and there is no empirical evidence of its failure. This strongly suggest that real physical objects are classical (meaning they do not display quantum properties) only approximatively. There are no *exactly* classical objects, strictly speaking, as everything we interact with is made of atoms and photons, which obey quantum theory.

It is unconvincing to use concepts valid only within an approximation when formulating the fundamental theory of nature. Therefore the attempts at interpreting quantum theory as a universal theory do not rely on postulating classical objects. This is the case for instance for the Everettian quantum mechanics [232, 254, 258] and the pilot wave theory [97, 118] and Relational Quantum Mechanics (RQM) [155, 221, 226]. The emergence of classical world (or worlds) has already been studied in both Everettian quantum mechanics and pilot wave theory. In this chapter, based on [85], we investigate how the classical world arises in RQM, namely, how some relative facts become stable thanks to certain physical interactions.

8.1.1 Relative facts

Relative facts are *defined* to happen whenever a physical system interacts with another physical system. While relative facts play a central role in RQM, their definition and their usefulness are independent of the interpretation. We shall discuss this role in detail in the next section.

Let us consider two systems $\mathcal{S}$ and $\mathcal{F}$. If an interaction affects a variable $L_{\mathcal{F}}$ of $\mathcal{F}$ in a manner that depends on the value of a certain variable $L_{\mathcal{S}}$ of $\mathcal{S}$, then the value of $L_{\mathcal{S}}$ is a fact relative to $\mathcal{F}$. That is, whenever a system $\mathcal{F}$ is affected by a variable of another system, the value of that variable becomes a fact for $\mathcal{F}$. This is true by definition irrespectively of whether $\mathcal{F}$ is a classical system. The interaction with $\mathcal{F}$ is the context in which that variable[1] takes a specific value; we call the system $\mathcal{F}$, in this role, a "context". The interaction with the context determines the fact that a certain variable has a value in that context.

Stable facts are a strict subset of the relative facts: there are many relative facts that are not stable facts. Quantum theory provides probabilities relating relative facts, but these satisfy (8.2) only if b and the a_i are facts relative to the *same* system. That is, if we label facts with their context (writing $a^{(\mathcal{F})}$ for a fact relative to system $\mathcal{F}$), then it is always the case that

$$P(b^{(\mathcal{F})}) = \sum_i P(b|a_i)P(a_i^{(\mathcal{F})}). \tag{8.3}$$

In contrast, whenever $\mathcal{W} \neq \mathcal{F}$, it is in general not the case that

$$P(b^{(\mathcal{W})}) = \sum_i P(b|a_i)P(a_i^{(\mathcal{F})}). \tag{8.4}$$

If (8.4) holds, we say that the value of the variable $L_{\mathcal{S}}$ is *stable for* $\mathcal{W}$.

The failure of (8.4) is easily understood in terms of the standard language of quantum theory: it is the presence of interference effects. If $\mathcal{F}$ is sufficiently isolated

[1] We use 'variable' to denote any quantity that in the classical theory is a function on phase pace. We prefer to avoid the expression 'observable' because it is loaded with irrelevant extra baggage: the ideas of observation and a complex observer. The term 'context' is used here in a sense similar to that in [119]. However we do not require the context to be classical. See [10].

it may be possible to maintain quantum coherence for the compound system $\mathcal{S}-\mathcal{F}$. The interaction entangles the two systems and interference effects between different values of the variable $L_{\mathcal{S}}$ can later be detected in the measurements by an observer $\mathcal{W}$. The probabilities for facts of the $\mathcal{S}-F$ system relative to $\mathcal{W}$ can indeed be computed from an entangled state of the form

$$c_1 \left|a_1\right\rangle \otimes \left|Fa_1\right\rangle + c_2 \left|a_2\right\rangle \otimes \left|Fa_2\right\rangle, \tag{8.5}$$

where a_i are values of $L_{\mathcal{S}}$ and Fa_i are values of $\mathcal{F}$'s "pointer variable" $L_{\mathcal{F}}$. Probabilities computed from this state violate (8.4) as they feature interference terms because what sums is amplitudes, not probabilities. The value of $L_{\mathcal{S}}$, therefore, is not a stable fact.

Hence, facts relative to a system $\mathcal{F}$ cannot in general be taken as conditionals for computing probabilities of facts relative to a different system $\mathcal{W}$. Equation (8.2) holds only if b and a_i are facts relative to the same system, but fails in general if used for facts relative to different systems.

While the notation $\mathcal{S}$ for 'system', $\mathcal{F}$ for 'Friend' and $\mathcal{W}$ for 'Wigner' is meant to evoke the famous *Wigner's friend* thought experiment [262] discussed in 3.2, in the discussion above there are no assumptions about the system $\mathcal{F}$ being quantum or classical, microscopic or macroscopic.

So, what exactly characterises a stable fact, among the relative facts? What gives rise to stable facts?

8.1.2 Decoherence

Since stability is a characteristic feature of the classical world, whose facts invariably satisfy (8.2), answering the questions above amounts to explaining in terms of relative facts what it takes for a system to be classical.

Various characterisations of a classical or semiclassical situation can be found in the literature: large quantum numbers, semiclassical wavepackets or coherent states, macroscopic systems, large or infinite number of degrees of freedom... All these features play a role in characterising classical systems in specific situations. But the key phenomenon that makes facts stable is decoherence [270, 273, 274]: the suppression of interference that happens when some information becomes inaccessible. Let us see in an example how this plays out.

Consider two systems $\mathcal{F}$ and $\mathcal{E}$ ($\mathcal{E}$ for "Environment"), and a variable $L_{\mathcal{F}}$ of the system $\mathcal{F}$. Let Fa_i be the eigenvalues of $L_{\mathcal{F}}$. A generic state of the compound system $\mathcal{F}-\mathcal{E}$ can be written in the form

$$\left|\psi\right\rangle = \sum_i c_i \left|Fa_i\right\rangle \otimes \left|\psi_i\right\rangle, \tag{8.6}$$

where $\left|\psi_i\right\rangle$ are normalised states of $\mathcal{E}$. Let us define

$$\epsilon = \max_{i\neq j} \left| \left\langle\psi_i|\psi_j\right\rangle \right|^2. \tag{8.7}$$

Now, suppose that: (a) ϵ is vanishing or very small and (b) a system $\mathcal{W}$ does not interact with $\mathcal{E}$. Then the probability $P(b)$ of any possible fact relative to $\mathcal{W}$

resulting from an interaction between $\mathcal{F}$ and $\mathcal{W}$ can be computed from the density matrix obtained tracing over $\mathcal{E}$, that is,

$$\rho = \mathrm{tr}_{\mathcal{E}} |\psi\rangle\langle\psi| = \sum_i |c_i|^2 |Fa_i\rangle\langle Fa_i| + O(\epsilon). \tag{8.8}$$

By setting $P(Fa_i^{(\mathcal{E})}) = |c_i|^2$, we can then write

$$P(b^{(\mathcal{W})}) = \sum_i P(b|Fa_i)P(Fa_i^{(\mathcal{E})}) + O(\epsilon). \tag{8.9}$$

Thus, probabilities for facts b relative to $\mathcal{W}$ calculated in terms of the possible values of $L_{\mathcal{F}}$ satisfy (8.4), up to a small deviation of order ϵ. Hence the value of the variable $L_{\mathcal{F}}$ is a fact relative to $\mathcal{E}$ that is stable for $\mathcal{W}$ *to the extent* to which one ignores effects of order ϵ. In the limit $\epsilon \to 0$, the variable $L_{\mathcal{F}}$ of the system $\mathcal{F}$ is exactly stable for $\mathcal{W}$.

Extensive theoretical work has shown that decoherence is practically unavoidable and extremely effective as soon as large numbers of degrees of freedom are involved [275]. The variables of $\mathcal{F}$ that decohere, namely the specific variables for which ϵ becomes small, are determined by the actual physical interactions between $\mathcal{F}$ and $\mathcal{E}$ (they are those variables that commute with the interaction Hamiltonian). The decoherence time, namely the time needed for ϵ to become so small that interference effects become undetectable by given observational methods, can be computed and is typically extremely short for macroscopic variables of macroscopic objects. All this is well understood. It is important for what follows to emphasise two subtle aspects of decoherence.

First, decoherence is not an *absolute* phenomenon, but a relational one: it depends on how the third system $\mathcal{W}$ interacts with the combined system $\mathcal{F}$–$\mathcal{E}$. This is because assumption (b) above is just as crucial as assumption (a) in deriving (8.9). Another system $\mathcal{W}'$ that interacts differently with $\mathcal{F}-\mathcal{E}$ might be able to detect interference effects.

Second, decoherence implies that an event regarding two systems $\mathcal{F}$ and $\mathcal{E}$ is stable for a *third* system $\mathcal{W}$. Hence, a fact stable for $\mathcal{W}$ is not necessarily a fact relative to $\mathcal{W}$. That is, the variable $L_{\mathcal{F}}$ is stable for $\mathcal{W}$ even if the latter has not interacted with it, so there is no fact relative to $\mathcal{W}$ yet. This is what allows one to say that, with respect to $\mathcal{W}$, the "state of the system $\mathcal{F}$ has collapsed into the state $|Fa_i\rangle$ with probability $P(Fa_i) = |c_i|^2$," even though $\mathcal{W}$ has not interacted with $\mathcal{F}$.

These observations show that decoherence does not imply that there is a perfectly classical world of absolute facts, although it does explain why (and when) we can reason in terms of stable, hence approximatively classical, facts.[2]

[2]There is a limit case in which a fact can be stable even in the absence of decoherence. This is when one of the amplitudes in (8.6), say c_1, is very close to 1. If $\mathcal{W}$ does not interact with $\mathcal{E}$, then the probabilities for facts relative to $\mathcal{W}$ can be computed using $\rho' = |Fa_1\rangle\langle Fa_1| + O(\eta)$, where $\eta = 1 - |c_1|^2$. In this case, one can reason as if the value of $L_{\mathcal{F}}$ were a fact relative to $\mathcal{W}$, up to order η effects. Thus, when a fact relative to a system has very high probability, then it is stable for other systems, because the interference effects are small. Notice the differences with the Einstein-Podolsky-Rosen [93] criterion for an 'element of reality:' the above only holds to the extent to which $\mathcal{W}$ cannot interact with $\mathcal{E}$, and there needs to be a fact relative to $\mathcal{E}$ in the first place.

8.1.3 Measurements

If two systems $\mathcal{S}$ and $\mathcal{F}$ interact so that their respective variables $L_\mathcal{S}$ and $L_\mathcal{F}$ get entangled, and if $L_\mathcal{F}$ is stable for $\mathcal{W}$, it follows immediately from the definitions that also $L_\mathcal{S}$ is stable for $\mathcal{W}$.

This is precisely what happens in a typical quantum measurement of a variable $L_\mathcal{S}$ in a laboratory. Thinking of $\mathcal{S}$, $\mathcal{F}$ and $\mathcal{W}$ as, respectively, the system being measured, the apparatus and the experimenter, we can separate the measurement in three stages:

1. An interaction between the system and the apparatus entangles $L_\mathcal{S}$ with a pointer variable $L_\mathcal{F}$ of the apparatus.

2. $L_\mathcal{F}$ gets correlated with a large number of microscopic variables (forming $\mathcal{E}$) that are inaccessible to the observer $\mathcal{W}$.

3. The observer $\mathcal{W}$ interacts with the pointer variable $L_\mathcal{F}$ to learn about $L_\mathcal{S}$.

Let's trace this same story in terms of relative facts:

1. A relative fact is established between $\mathcal{S}$ and $\mathcal{F}$.

2. A relative fact is established between $\mathcal{F}$ and $\mathcal{E}$. Since $\mathcal{W}$ does not interact with $\mathcal{E}$, this stabilises the previous fact for $\mathcal{W}$.

3. A relative fact is established between $\mathcal{F}$ and $\mathcal{W}$. This has consequences on $\mathcal{W}$'s future interactions with $\mathcal{S}-\mathcal{F}$.

Already at stage 2, the observer can apply (8.4) since the interaction with the inaccessible degrees of freedom greatly suppresses interference terms. The observer might say "$L_\mathcal{S}$ has been measured," and assume that the pointer of the apparatus moved one way or the other. In the mathematical formalism, $\mathcal{W}$ can assume that "$\mathcal{S}$'s wavefunction has collapsed." Note however that neither the value of $L_\mathcal{S}$ or $L_\mathcal{F}$ is a fact for $\mathcal{W}$ at this stage. Stability simply allows $\mathcal{W}$ to "de-label" facts relative to $\mathcal{F}$. It is is only at stage 3 that the value of $L_\mathcal{F}$ becomes a fact for $\mathcal{W}$. Note that the value of $L_\mathcal{S}$ is still not a fact relative to $\mathcal{W}$, but it is a stable fact for $\mathcal{W}$. However, based on the value of $L_\mathcal{F}$, the experimenter $\mathcal{W}$ can update the state for $\mathcal{S}$. The experimenter can reason as if $L_\mathcal{S}$ took the value that she read on the apparatus' pointer variable.

It is the way that the four systems $\mathcal{S}$, $\mathcal{F}$, $\mathcal{W}$, and $\mathcal{E}$ couple to each other that makes $\mathcal{F}$ a measuring apparatus for $\mathcal{W}$. The stability of $\mathcal{F}$ for $\mathcal{W}$ extends to all other variables that interact with $\mathcal{F}$, hence $\mathcal{W}$, on might say that "$\mathcal{F}$ causes $\mathcal{S}$ to collapse." But, in fact, this "collapse" is not objective, it is relational and effective. Another system $\mathcal{W}'$ that couples differently to these systems might still be able to detect interference effects.

In summary, we can distinguish two notions of facts that play a role in quantum theory: relative facts and stable facts. Quantum theory allows us to talk about relative facts and compute probabilities for them. Equation (8.3) holds but (8.4) does not. The violation of (8.4) is quantum interference.

Stable facts are a subset of the relative facts. They satisfy (8.4). A relative fact about a system $\mathcal{F}$ is stable for a system $\mathcal{W}$ if $\mathcal{W}$ has no access to a system $\mathcal{E}$ which is sufficiently entangled with $\mathcal{F}$. Stability is only approximate and relational. Approximate, because no fact is exactly stable for any finite ϵ. Relational, because it depends on how the putative 'observer' system couples to the system and the environment.

8.2 Facts and reality

We have given definitions of relative and stable facts, and studied their properties. In this section we discuss the roles of relative and stable facts for the interpretation of quantum theory, namely for the relation between the formalism and the reality it describes.

8.2.1 The link between the theory the world

Let us compare advantages and difficulties of interpreting either stable or relative facts as the link between theory and reality.

Stable facts are taken as the link between the formalism and the world in textbook interpretations of quantum theory. They are the conventional "measurement outcomes" in a macroscopic laboratory. They are similar to the facts of classical mechanics because, in the world described by classical mechanics, all facts (variables having certain values at certain times) are exactly stable: the (epistemic) probabilities for them to happen are *always* exactly consistent with (8.2). In quantum mechanics, facts stable for us humans are ubiquitous because of the ubiquity of decoherence and the frequency of interactions.

There are however two difficulties in taking stable facts as the basis of the quantum ontology. First, stability is relational. Facts are stable only for a system that does not have sufficiently precise interactions with an environment system. The system and environment are still in a superposition with respect to a third system. Therefore one does not avoid relationalism by restricting to stable facts. Second, more seriously, stability is only approximate in general. At no point the interference terms perfectly vanish. These are serious difficulties if we want to take stable facts as the only primary elements of reality. How stable does a fact need to be before it is real? And with respect to what systems does it have to be stable, in order to be real? Any answer to these questions is bound to be as unsatisfactory as the textbook interpretation that requires a classical world. The alternative is to embrace the contextuality of the theory in full, and base its ontology on all relative facts.

Relative facts form the basis of a realist interpretation in Relational Quantum Mechanics (RQM). The fundamental contextuality that characterises quantum theory is interpreted in RQM as the discovery that facts about a system are always defined relative to another system, with which the first system interacts. In the early history of quantum theory it was recognised that every measurement involves an interaction, and it was said that variables take values only upon measurement. RQM notices that every interaction is in a sense a measurement, in that it results in the value of a variable to become a fact. These facts are not absolute, they belong to a context. And there is no 'special context': any system can be a context for any other system.

The quantum state ("the wavefunction") does not have an ontic interpretation in RQM. The state is not a "thing", nor a condition of a system. Rather, it is what a physicist uses to calculate probabilities for relative facts between physical systems to happen, given the relevant information she has. It follows that RQM has no use for a "wavefunction of the universe" that forever evolves unitarily, as this would be a tool to calculate probabilities of facts relative to something that does not exist: a system that is not part of the universe.

Unlike other epistemic interpretations of quantum theory in which the wavefunction plays only an epistemic role, the ontology of RQM is realist in the sense that it is not about agents, beliefs, observers, or experiences: it is about real facts of the world and relative probabilities of their occurrence. The ontology is relational, in the sense that it is based on facts established at interactions and are labelled by physical contexts. Relative facts, therefore, provide a relational but realist interpretation to quantum theory which does not need to refer to complex agents.

8.2.2 No-go theorems for absolute facts

A number of results have recently appeared in the literature as no-go theorems for absolute (non-relative) facts [32, 36, 103]. These results analyse the extended Wigner's friend scenario (EWFS), in which instead of a superobserver reasoning about his friend in a hermetically sealed laboratory, there are two superobservers each reasoning about their own friend, with the friends entangled.

Quantum theory cannot consistently describe the use of itself

In [103], Frauchiger and Renner use a EWFS to show that quantum theory is inconsistent under a certain number of assumptions. A key assumption used to derive the contradiction is the absolute nature of facts. This is assumption (C) in the paper, which can be stated as follows: "If $\mathcal{W}$, applying quantum theory, concludes that $\mathcal{F}$ knows that $L_\mathcal{S} = a$, then $\mathcal{W}$ can conclude that $L_\mathcal{S} = a$." The 'C' stands for consistency: the authors argue that this assumption is required to deem the theory consistent: different agents using the same theory must arrive at the same conclusions. From the point of view of this chapter, this is an ironic choice of name, as it is precisely this assumption that leads to contradiction according to RQM.

In terms of relative facts, assumption (C) implies: "If $\mathcal{W}$, applying quantum theory, can be certain that $L_\mathcal{S} = a$ relative to $\mathcal{F}$, then $\mathcal{W}$ can reason as if $L_\mathcal{S} = a$ also relative to $\mathcal{W}$." Now, as we have shown, this holds only if every fact relative to $\mathcal{F}$ is stable for $\mathcal{W}$, which is not guaranteed and depends on the physics. Therefore Assumption (C) only holds if $\mathcal{S}$ or $\mathcal{F}$ decohere with respect to $\mathcal{W}$. In the Frauchiger and Renner protocol, the superobservers are supposed to have full quantum control on their friends and the contents of their labs. Thus, by definition, what is stable for the friends is not stable for $\mathcal{W}$. Hence, the contradiction follows from inappropriately mixing contexts: forgetting that facts are relative and therefore (8.4) does not hold in general.

Indeed, as pointed out in [272] and worked out in detail in [214, 215], no contradiction can be derived if one additionally assumes that what is decoherent for the friends (the laboratory they are in) is also decoherent for $\mathcal{W}$. As a side note, the analysis of how an agent should reason about an experiment that will be performed on him has not been done within RQM yet. The reader is invited to consider the analysis within QBism [50, 77], since also QBism holds that assumption (C) fails and quantum states are only used to calculate probabilities from the point of view of a given system.

A strong no-go theorem on the Wigner's friend paradox

Another enlightening result is the recent Bong *et. al.* [32], which is a strengthening of a previous result by Brukner [36, 212]. In [32], the authors show that the conjunction of (a) absoluteness of observed events, (b) no superdeterminism and (c) locality imply that correlations in the extended Wigner's friend scenario must satisfy some inequalities, called the Local Friendliness (LF) inequalities. Like Bell's inequalities[3] [16, 19], these are derived in a theory-independent way. The authors then show that quantum theory predicts the violation of these inequalities. Thus the universal validity of quantum theory implies that one of the three properties above does not hold.

The word 'locality' means different things in different physics communities. The notion used to derive the LF inequalities is the one that Bell used to derive his inequalities in [16]. In operational language, (c) says that a free choice does not alter the probabilities of a spacelike separated event. Most epistemic interpretations accept this notion of locality [47, 52], while it is rejected by the pilot-wave interpretation [118]. No superdeterminism simply means that free choices are possible so that, in particular, the measurement settings can be chosen so as to be uncorrelated with other relevant variables. See [264] for an in depth analysis on the notions of locality and superdeterminism in the context of the implications of Bell's theorems.

If one believes that quantum theory holds at arbitrary scales, wishes to maintain locality and reject superdeterminism, one has no choice but to reject the absoluteness of observed events. Absoluteness of observed events means that if $\mathcal{W}$ deems that $\mathcal{F}$ is an observer, then $\mathcal{W}$ can use (8.4)—even if $\mathcal{W}$ has full quantum control on $\mathcal{F}$. This clearly does not hold in RQM: if $\mathcal{W}$ has full quantum control on $\mathcal{F}$ then facts relative to $\mathcal{F}$ are not stable for $\mathcal{W}$ and thus (8.4) does not hold. Note that in RQM there are no special 'observer' systems so an 'observed event' is simply a fact relative to a given system.

Remarkably, the LF inequalities have already been experimentally violated when the friends are single photons [32]. One might be tempted to dismiss the results on the ground that photons do not generate facts ("photons are not observers"), but this opens the problem of deciding which systems give rise to facts. If quantum theory is universally valid, advances in quantum technologies will allow to perform the same experiment with increasingly complex "friends". The predictions of quantum theory remain the same: the statistics are incompatible with the assumptions of absolute facts.

The violation of the LF inequalities is no way in tension with the relational interpretation. The opposite is true: the result is taken as evidence that the facts quantum theory deals with are facts relative to systems.

[3] For a discussion of the implications of the ontology of RQM to Bell's inequalities, see [154, 244] and [169, 204].

8.2.3 Conclusions and final comments

The insight of Relational Quantum Mechanics (RQM) is that recognising the relative nature of facts offers a straightforward solution to the measurement problem. The measurement problem is the apparent incompatibility between two postulates: the "projection" and the "linear evolution" postulate. Both postulates can be correct: they refer to facts relative to different systems. Say that $\mathcal{S}$ interacts with $\mathcal{F}$, so that a fact relative to $\mathcal{F}$ is established. Then the projection postulate is used to update the state of $\mathcal{S}$ with respect to $\mathcal{F}$, while the unitary evolution postulate is used to update the state of $\mathcal{S}-\mathcal{F}$ with respect to a third system $\mathcal{W}$.

In a slogan: "Wigner's facts are not necessarily his Friend's facts".
This by no means implies that when Wigner and his friend compare notes they find contradictions [221]. Interactions between $\mathcal{S}$ and $\mathcal{F}$ do have influence on the facts relative to $\mathcal{W}$. Indeed, after an interaction, $\mathcal{S}$ and $\mathcal{F}$ are entangled relative to $\mathcal{W}$, meaning that in interacting with the two systems, $\mathcal{W}$ will find the two correlated. Therefore Wigner will always agree with his Friend about the value of $L_{\mathcal{S}}$ once he too interacts with them. In this sense, relative facts correspond to real events, they have universal empirical consequences.

Still, accepting the relativity of all facts is a strong conceptual step. It amounts giving up the absolute nature of facts, namely, the existence of an absolute "macro-reality" in the language used in discussions of Bell's inequalities [264]. Such a macroreality only emerges approximately, relative to systems for which decoherence is sufficiently strong.

Decoherence has always played a peculiar role in the discussions on the measurement problem. On the one hand, it is simply a true physical phenomenon, obviously relevant for shedding light on quantum measurement. On the other hand, there is consensus that decoherence alone is not a solution of the measurement problem, because it does not suffice to provide a link between theory and reality. Decoherence needs an ontology. Relative facts provide such a general ontology, which is well defined with or without decoherence. Decoherence clarifies why a large class of relative facts are stable for us and thus form the stable classical world we live in.

The violation of (8.2) when used for facts relative to different systems sheds also some light on the underpinnings of quantum logic. The violation of (8.2), indeed, has been interpreted as a violation of classical logic [28], as it can be written as

$$P(b \text{ and } (a_1 \text{ or } a_2)) \neq P((b \text{ and } a_1) \text{ or } (b \text{ and } a_2)), \tag{8.10}$$

in contradiction with the classical logic theorem

$$b \text{ and } (a_1 \text{ or } a_2) = (b \text{ and } a_1) \text{ or } (b \text{ and } a_2). \tag{8.11}$$

The apparent violation of logic is understood in RQM as a result of forgetting that facts are relative: labelled by a context, as Bohr has repeatedly pointed out. Facts relative to a context cannot be used, in general, to compute probabilities of facts related to other contexts because what is a fact in a certain context is not necessarily a fact in other contexts.

As a final remark, observe that if the quantum state has no ontic interpretation, the only meaning of "being in a quantum superposition" is that interference effects are to be expected. Saying "Friend is in a quantum superposition" does not mean anything more than saying that Wigner would be mistaken in using (8.4). It has no implications on how Friend would "feel" while being in a superposition. Friend sees a definite result of her measurement, a fact, and this does not prevent Wigner from having the chance to see an interference effect in *his* facts. Wigner's friend does not stop being an observer simply because Wigner has a chance to detect interference effects in *his* facts. Schrödinger's cat has no reason to feel "superposed".

Chapter 9

Consequences of the relativity of facts

In this chapter, based on [84], we wish to discuss the points raised by two recent papers, one by Jacques Pienaar [205] and one by Časlav Brukner [38]. These authors present insightful observations and objections on the Relational interpretation of Quantum Mechanics (RQM). We point out that the observations in them are not challenges against RQM: they are arguments that clarify and sharpen some aspects of this interpretation.

Pienaar separates his objections to the relational interpretation into two parts, the first regards the analogy between RQM and special relativity, the second regards the status of objectivity in RQM. In the first part, Pienaar points out that the analogy with special relativity is only partial: the sense in which variables are "relative" in special relativity is more restricted than the sense in which variables are "relative" in RQM. In the second part, he argues that RQM cannot be reduced to the relativity of variables, because facts themselves are relative, and there is no absolute way of comparing the perspectives of two systems.

Both observations are correct, but they are not objections to RQM. They are considerations that emphasise the radicality of the RQM perspective. The relational interpretation does not pretend to make quantum theory less revolutionary than what it is. It only claims that there exists a coherent and complete way of thinking about quantum phenomena that makes sense without requiring many worlds, hidden variables, cognitive agents, or a macroscopic classical world. Hence the two objections by Pienaar are only objections to the hope to spoil RQM of its core (radical) idea.

Pienaar makes his objections concrete in the form of five no-go theorems that are supposed to pitch the claims of RQM against one another. To do so, he summarises RQM in terms of six "key claims" **RQM:1–6**. This is a detailed and mostly accurate account of RQM. But it contains one misstep: a misrepresentation of the claims **RQM:5** and **RQM:6** (see below). This misrepresentation is common to both [38] and [205], and regards the meaning of the quantum state. In RQM, the quantum state is not a representation of reality: it is always a relative state and is only a mathematical tool used to compute probabilities of events *relative to a given system*. The quantum state of a composite system relative to an external system is not an account or record of relative events *between the subsystems* of the composite system.

Assuming that the quantum state is more than this is a misunderstanding leading to the apparent contradictions. This same mischaracterisation of RQM undermines Brukner's critique in [38]. Brukner's theorem then does not appear as a critique of RQM. It becomes instead a restriction on the concept of knowledge—concept that plays no *fundamental* role in the formulation of RQM.

Because of the mischaracterisation of **RQM:5** and **RQM:6**, and the consequent over-emphasis on the quantum state, the theorems, as we shall see, either fall apart or become evidence of the *consistency* of the interpretation. Pienaar's and Brukner's acute arguments actually turn out to illuminate and emphasise the consistency of the interpretation, rather than challenging it.

In section 9.1, we comment on Pienaar's formulation RQM's claims, pointing out where it is imprecise. We also briefly anticipate how each of the five no-go theorems is resolved in a proper understanding of RQM. In section 9.2, we comment on the relativistic analogy and, in section 9.3, we address Pienaar's comments about objectivity in RQM. In this context, we present also a general philosophical consideration regarding the physical meaning of a subject's knowledge. In section 9.4, we respond to Brukner's paper.

As in the previous chapter, we will often consider three interacting systems $\mathcal{W}$, $\mathcal{F}$ and $\mathcal{S}$, and describe the events relative to either $\mathcal{F}$ or $\mathcal{W}$. The notation is meant to suggest the setup of Wigner's friend thought experiment [262], although no assumption about these systems being conscious observers or decision-making agents is necessary.

9.1 RQM's key claims

Pienaar summarises the RQM literature in terms of six claims, reported in full for reference:

RQM: 1. Any system can be an observer. Any physical system can play the role of an observer in a physical interaction.

RQM: 2 No hidden variables. Any variable that exists in the observer's causal past and which is relevant to predictions about future quantum events relative to the observer must be a quantum event contained in their perspective.

RQM: 3 Relations are intrinsic. The relation between any two systems $\mathcal{A}$ and $\mathcal{B}$ is independent of anything that happens outside these systems' perspectives. In particular, the state of $\mathcal{B}$ relative to $\mathcal{A}$ depends only upon $\mathcal{A}$'s observation of $\mathcal{B}$ and $\mathcal{A}$'s past history of interactions (similarly for the state of $\mathcal{A}$ relative to $\mathcal{B}$).

RQM: 4 Comparisons are relative to one observer. It is meaningless to compare the accounts of any two observers except by invoking a third observer relative to which the comparison is made.

RQM: 5 Any physical correlation is a measurement. Suppose an observer measures a pair of systems and thereby assigns them a joint state which exhibits perfect correlations between some physical variables. Then the two systems have measured each other (entered into a measurement interaction) relative to the observer, and the physical variables play the roles of the 'pointer variable' and 'measured variable' of the systems.

RQM: 6 Shared facts. In the Wigner's friend scenario, if $\mathcal{W}$ measures $\mathcal{F}$ to 'check the reading' of a pointer variable (i.e. by measuring $\mathcal{F}$ in the appropriate 'pointer basis'), the value he finds is necessarily equal to the value that $\mathcal{F}$ recorded in her account of her earlier measurement of $\mathcal{S}$.

This is a good summary of RQM, but some points are slightly misleading, and one is strongly misleading. Let us comment on each claim.

RQM:1 Any system can be an observer is essentially correct but poorly phrased, because of the term "observer". As we have seen in the previous chapter, RQM distinguishes relative facts from stable facts. Relative facts (or "events") form the basis of the ontology; they are ubiquitous and do not require any special property of the physical systems involved in order to happen. Stable facts are facts stabilised by decoherence, in the sense that their relativity can be ignored by a large class of systems. It is better to reserve the use of operational expressions such as "observer" and "measurement" to those specific situations where there is enough decoherence to underpin stability, for instance, when there is a scientist making observations, or a macroscopic system storing memory.[1]

Terminology aside, the actual content of **RQM:1** is correct, namely, we assume something can happen relative to any system—not only measuring apparata or "observers" that are special in any sense. So we would rephrase this claim as:

RQM: 1⋆ Events (facts) can happen relative to any physical system. Events happen in interactions between any two systems and can be described as the actualisation of the value of a variable of one system relative to the other.

RQM:2 No hidden variables is a statement about the universality of QM. It is correct, but RQM is consistent with the time-reversal invariance of fundamental physics (see chapter 7), and thus the formulation given by Pienaar must be generalised: it remains valid when swapping 'past' and 'future'. **RQM:3 Relations are intrinsic** also does not require any modification.

RQM:4 Comparisons are relative to one observer is another key tenet of RQM. The idea is that contradictions arise when trying to equate descriptions of physics in two different contexts, namely relative to different systems. This is for instance what happens in the Frauchiger and Renner experiment [103], as we have argued in chapter 8. We rephrase this claim in a cleaner language as:

RQM: 4⋆ Comparisons are only relative to a system. It is meaningless to compare events relative to different systems, unless this is done relative to a (possibly third) system.

The point is that comparisons can only be made via a (quantum-mechanical) interaction. In the Wigner's friend setup, $\mathcal{W}$ might compare the result of his measurement on $\mathcal{S}$ with that of $\mathcal{F}$ only by *physically* interacting with $\mathcal{F}$ in an appropriate manner.

[1] To be sure, the RQM literature does use the operational terminology ambiguously and it is indeed common to call "observer" any system with respect to which a variable takes values. We shall also indulge in this abuse of language below.

There is no meaning in comparing facts relative to $\mathcal{W}$ with facts relative to $\mathcal{F}$, (or relative to Schr ́odinger and his cat) apart from this direct physical interaction.

We now come to the troublesome points. **RQM:5 Any physical correlation is a measurement** is the main problem with Pienaar's account. In RQM, facts determine states, not the other way around. Knowing the state of a system $\mathcal{S}$ is not sufficient to deduce the set of facts relative to the subsystems of $\mathcal{S}$. Attempting to do so leads to contradictions, as Pienaar's theorems 3 and 5 show.

The problem here is what determines what. Pienaar and Brukner take the state as primitive and assume that out of the state one can deduce which events happen in a composite system. This is not RQM. In RQM, it is the other way around. Events are primitive. Their happening is partially reflected in the state of the composite system relative to a third system. But only partially. Events cannot be read out of the state. The existence of a correlation between two variables gives indications about events, but in general it is not sufficient to tell which event was or was not realised. To know what event lead to the creation of a correlation, one needs to know more, for example the dynamics that coupled the two systems and, in particular, what variables are involved in the interaction.

Besides this key misrepresentation, there is also a terminological problem in **RQM:5**, parallel to the one pointed out for **RQM:1**. Pienaar calls a "measurement" what the RQM literature calls an event that establishes a fact. It is much better to reserve the loaded expression "measurement" to interactions that stabilise certain facts and require decoherence.

A proper reformulation of **RQM:5**, is:

RQM:5⋆ An interaction between two systems results in a correlation within the interactions between these two systems and a third one. With respect to a third system $\mathcal{W}$, the interaction between the two systems $\mathcal{S}$ and $\mathcal{F}$ is described by a unitary evolution that potentially entangles the quantum states of $\mathcal{S}$ and $\mathcal{F}$.

As we shall see, while **RQM:5** is in tension with **RQM:3**, **RQM:5⋆** is not. Note also how**RQM:5⋆** goes hand in hand with **RQM:1⋆**. These two assumptions together provide the resolution of the measurement problem in RQM. Von Neumann measurements are compatible with unitary evolution because they describe facts relative to two interacting systems ($\mathcal{S}$ and $\mathcal{F}$) while the unitary evolution regards facts relative to a third system ($\mathcal{W}$).

Finally, **RQM:6 Shared facts** as stated by Pienaar is either wrong (if it is intended to override **RQM:4**) or a tautology. It is not possible to decide which because Pienaar does not mention the context of the comparison. According to **RQM:4**, the *only* meaning of a comparison between an event relative to $\mathcal{F}$ and an event relative to $\mathcal{W}$ is in the context of a measurement made by a specified system.

A non ambiguous claim is:

> **RQM:6⋆ Shared facts.** In the Wigner's friend scenario, if $\mathcal{W}$ measures $\mathcal{S}$ on the same basis on which $\mathcal{F}$ did, then appropriately interacts with $\mathcal{F}$ to 'check the reading' of a pointer variable (i.e. by measuring $\mathcal{F}$ in the appropriate 'pointer basis'), the two values found are in agreement.

We briefly anticipate the resolution of the no-go theorems, discussed in detail in the following sections.

- Theorem 1 does not bite because it relies on Pienaar's version of **RQM:5**.
- Theorem 2 relies on two assumptions that are not valid in RQM because they misrepresent the role of the quantum state in the interpretation.
- Theorem 3 relies on **RQM:6**, which is incorrect.
- Theorem 4 does not bite because of **RQM:5** again.
- Theorem 5 relies on **RQM:5**, which is incorrect.

Theorems 2, 3, and 5 offer two alternatives (two 'horns'). As we shall discuss below, RQM 'grabs a horn' in each of them. Theorem 2 elucidates what RQM is about, while grabbing the horn in theorems 3 and 5 simply amounts to correcting Pienaar's mischaracterisation of RQM. Theorems 1 and 4 do not apply to RQM, for the same important reason, they are based on the misunderstanding of the role of the quantum state.

9.2 The analogy with relativity

The analogy between special relativity and relational quantum mechanics is often used in presentations of the latter. Pienaar shows in detail that the relationalism on which RQM is based is far more radical that the relationalism that underpins classical relativity. Therefore the conceptual novelty of quantum theory cannot be reduced to a simple recognition that all variables are relative, like velocity is relative in mechanics. He characterises[2] the relationality of RQM with the slogan "facts are relative," which is also correct, as we have seen in the previous chapter. On the other hand, Pienaar's claim that "without the conceptual analogy to classical relativistic relations, RQM would lose its core motivation as an interpretation" is too strong. The interest and the value of RQM does not depend on it being analogous to something else. As any interpretation of quantum mechanics, it derives its worth from the extent it elucidates our quantum world.

In addition, there are two other aspects of the analogy, that Pienaar disregards. First, special relativity is a conceptual advance based on the realisation that a previously "obvious" notion—absolute simultaneity—is in fact inappropriate to describe the world. RQM is a conceptual advance based on the realisation that another previously "obvious" notion—absolute facts—may in fact be inappropriate to describe the world. (We might soon have empirical evidence for this, see [32].) Second, there is a *methodological* similarity between RQM and special relativity: the idea of searching for transparent physical principles from which the mathematical structure of the theory can be defined. The two principles proposed in the first paper on RQM [221], are

[2] He suggests changing the name of the interpretation to 'Relative-facts interpretation of Quantum Mechanics'. That might be appropriate, but 'Relational' also works, because reality relative to one system—the collections of facts relative to that system—is composed of direct interactions this system has with the rest of the world. Rendering facts relative is a generalisation of relativity, albeit a drastic one.

1. The relevant information that can be extracted from a finite region of the phase space of a physical system is finite,
2. It is always possible to extract novel relevant information from a physical system.

are based on the idea that the theory describes the relative information that a system can gather about another system. As we have seen in section 3.1, these two principles serve as the first two axioms of Höhn's and Wever's compelling reconstruction [130, 132, 133].

In brief, the analogy with relativity played a historical role in the development of RQM and has some interest despite the fact that it is not complete. If the literature on RQM has given the impression that the radical conceptual novelty of quantum mechanics could be reduced to nothing else than some minor extension of special relativity, this was a mistake. RQM is genuinely radical.

Let us now look at the two theorems with which Pienaar supports his claims.

No-go theorem 1

> **Dilemma:** Suppose a system $\mathcal{F}$ has measured $\mathcal{S}$, and this fact is verified by a third system $\mathcal{W}$ who measures $\mathcal{F}$-$\mathcal{S}$. Then there exist situations in which one of the following must be true:
> (i) $\mathcal{F}$ has measured $\mathcal{S}$ simultaneously in incompatible bases, relative to $\mathcal{W}$;
> (ii) The basis in which $\mathcal{F}$ has measured $\mathcal{S}$ is indeterminate relative to $\mathcal{W}$.

Pienaar understands this dilemma to be a no-go theorem because both alternative (i) contradicts one of the main features of quantum mechanics, while (ii) contradicts **RQM:5**. The solution of the difficulty is that (ii) is correct and does not contradict any of the RQM claims, because it is **RQM:5**$\star$ and not **RQM:5** that characterises RQM and (ii) is not in contradiction with **RQM:5**$\star$. Underlying this, there is a misunderstanding of the role of the quantum state in RQM. Let us see this in more detail.

In the proof of the dilemma, a situation is considered in which the state of $\mathcal{S}-\mathcal{F}$ relative to $\mathcal{W}$ is

$$|\Psi\rangle_{\mathcal{SF}} = \sum_i \alpha_i \, |x_i\rangle_{\mathcal{S}} \, |Fx_i\rangle_{\mathcal{F}} \,, \tag{9.1}$$

where $\{x_i\}$ and $\{Fx_i\}$ denote eigenvalues of some observables X and F_X of $\mathcal{S}$ and $\mathcal{F}$ respectively. Pienaar notes that in general this Schmidt decomposition is not unique, and one could find other observables Y and F_Y such that

$$|\Psi\rangle_{\mathcal{SF}} = \sum_n \beta_n \, |y_n\rangle_{\mathcal{S}} \, |Fy_n\rangle_{\mathcal{F}} \,. \tag{9.2}$$

He then uses of **RQM:5** (every correlation is a measurement) to derive horn (i) of the dilemma. Since there is a correlation both between X and F_X and between Y and F_Y, then, allegedly, $\mathcal{F}$ has measured $\mathcal{S}$ simultaneously on the incompatible bases X and Y. This is not the case in RQM, **RQM:5** cannot be applied. All that $|\Psi\rangle_{\mathcal{SF}}$ tells us is which kinds of correlations exist between the variables of the two systems, relative to W.

The confusion arises also because of Pienaar's use of the word 'measurement'. Relative to $\mathcal{W}$, the only meaning that can be ascribed to the question of whether or not $\mathcal{F}$ has "measured" $\mathcal{S}$ is whether there is a correlation between the relevant variables of the two systems. Since there is a correlation between different pairs of variables, *in this sense and only in this sense*, the "measurement" happened in multiple bases. The strangeness of the statement is only the inappropriate use of the expression "measurement" in this situation. If we use proper expressions, everything returns to reasonable. With respect to $\mathcal{W}$, is there a correlation between variables of $\mathcal{S}$ and variables of $\mathcal{F}$? Yes there is. In which basis? In more than one basis.

So, how do we know which of $\mathcal{S}$'s variables became definite relative to $\mathcal{F}$? We do not, if we only know the state $|\Psi\rangle_{\mathcal{SF}}$. More information can be obtained from the dynamics of the system. The state (9.1) for instance may arise as a result of an interaction between $\mathcal{S}$ and $\mathcal{F}$ in which the evolution of $\mathcal{F}$ depends on the value of the variable X of $\mathcal{S}$. For example, the interaction Hamiltonian can depends on this variable. From the perspective of $\mathcal{F}$, this interaction leads to the actualisation of the variable X of $\mathcal{S}$. But the same state relative to $\mathcal{W}$ could arise via an interaction Hamiltonian that depends on the variable Y, and then it is this variable that actualises relative to $\mathcal{F}$. The physics of the two processes is different, but results in the same state relative to $\mathcal{W}$, namely in the same probability distribution of events relative to $\mathcal{W}$. The final state relative to $\mathcal{W}$ lacks information about what happens among subsystems.

Pienaar also refers to the observable M of the combined system $\mathcal{S} - \mathcal{F}$ that was introduced in [222]. This is an observable that $\mathcal{W}$ can measure to check the existence of a perfect correlation between certain variables:

$$M\, |x_i\rangle_{\mathcal{S}}\, |Fx_j\rangle_{\mathcal{F}} = \delta_{ij}\, |x_i\rangle_{\mathcal{S}}\, |Fx_i\rangle_{\mathcal{F}}\,. \tag{9.3}$$

The same M can be expressed as

$$M\, |y_n\rangle_{\mathcal{S}}\, |Fy_m\rangle_{\mathcal{F}} = \delta_{nm}\, |x_n\rangle_{\mathcal{S}}\, |Fx_m\rangle_{\mathcal{F}}\,. \tag{9.4}$$

Measuring $M = 1$ tells us that the correlation exists and is maximal. This is compatible with *either* X *or* Y having taken a definite value relative to $\mathcal{F}$. The value of M on its own, does not allow $\mathcal{W}$ to know which variable is definite relative to $\mathcal{F}$.

The central idea of RQM is that, since the only way for $\mathcal{W}$ and $\mathcal{F}$ to communicate is via a quantum mechanical measurement, there is no meaning to any other form of relations between the two. Here Pienaar is equating two distinct statements: (i) a variable of $\mathcal{S}$ has a value with respect to $\mathcal{F}$, and (ii) with respect to $\mathcal{W}$, there is a correlation to be expected between a variable of $\mathcal{S}$ and a pointer variable of $\mathcal{F}$. The first implies the second, but the second does not imply the first. The second can regard multiple bases even while the first cannot.

No-go theorem 2

This theorem expresses a contradiction between a set of three assumptions (i)-(iii) constraining the set of possible states that two systems $\mathcal{F}$ and $\mathcal{W}$ might assign to a third system $\mathcal{S}$ and the fact (iv) that not all state assignments are good states assignments. We report here the two relevant assumptions:

> (ii) Any valid state assignment $|\psi\rangle_{\mathcal{S}}$ by $\mathcal{F}$ can always be verified by $\mathcal{W}$. That is, there must exists a 'pointer basis' of $\mathcal{F}$ such that, if $\mathcal{W}$ were to measure in this basis and condition on the outcome, there would be a nonzero probability of updating the state of $\mathcal{S}$ relative to $\mathcal{W}$ to $|\psi\rangle_{\mathcal{S}}$.
> (iii) Conversely, any assignment $|\psi\rangle_{\mathcal{S}}$ by $\mathcal{F}$ which can be verified by $\mathcal{W}$ (in the above sense) must be a valid possible assignment for $\mathcal{F}$.

Then:

> **Dilemma:** The set of assumptions (i)-(iii) are together incompatible with (iv). Specifically, given that $\mathcal{W}$ assigns an entangled state [$|\Psi\rangle_{\mathcal{SF}}$] of the form [(9.1)], and assuming the coefficients α_i are all nonzero, then *every* pure state in the Hilbert space of $\mathcal{S}$ is a possible state relative to $\mathcal{F}$.

RQM resolves this no-go theorem by rejecting assumptions (ii) and (iii).

Again, the point is the role of the quantum state in RQM. The state does not represent a description of reality; it is a mathematical tool to compute the likelihood of events. Say $\mathcal{W}$ assigns state (9.1) to $\mathcal{S}-\mathcal{F}$ and then measures F_X and finds the value Fx. Then $\mathcal{W}$ will have to update the quantum state of $\mathcal{S}-\mathcal{F}$ to $|x\rangle_{\mathcal{S}}|Fx\rangle_{\mathcal{F}}$. In no way is $\mathcal{W}$ allowed to conclude that $\mathcal{F}$ had assigned the state $|x\rangle_{\mathcal{S}}$ to $\mathcal{S}$. For $\mathcal{W}$ to conclude that the new state of $\mathcal{S}$ relative to them is the state that $\mathcal{S}$ had assigned, $\mathcal{W}$ would need to know that the variable X had become a fact relative to $\mathcal{F}$.

Let us be even more explicit, and consider the original Wigner's friend thought experiment, where $\mathcal{F}$ is an actual human in a lab and the operational talk of the previous paragraphs can be understood literally. Wigner knows that Friend measures a qubit on the computational basis, and that the value of Z is then a fact relative to Friend. Wigner assigns a state proportional to

$$|0\rangle_{\mathcal{S}}|F0\rangle_{\mathcal{F}}+|1\rangle_{\mathcal{S}}|F1\rangle_{\mathcal{F}} \tag{9.5}$$

to the combined system. If Wigner then measures the Friend on the F_Z basis and obtains $F0$, he is allowed to conclude that F had assigned the state $|0\rangle$ to $\mathcal{S}$. What happens instead if Wigner decides instead to measure Friend on the complementary basis $\{|F\pm\rangle \propto |F0\rangle \pm |F1\rangle\}$ and obtains $F+$? Despite his experimental genius, he would be a fool to entertain that Friend had assigned the state $|+\rangle$ to $\mathcal{S}$! Wigner's choice of measuring on this complementary basis meant he had to forsake the ability to reveal Friend's assignment.

How radical is radical?

One point in Pienaar's rhetoric is to emphasise the radical relationalism of quantum phenomenology contrasting it with the consistency of the classical world. For instance, Pienaar writes:

> when two observers are in a situation where they disagree about the state of a system in RQM, the state relative to one observer *places no non-trivial constraints on the state relative to the other observer*, in stark contradistinction to disagreements about velocity and other classical quantities in relativity.

The misleading aspect of this rhetoric is that it ignores the physical source of the classical consistency. Classical consistency is not incompatible with quantum physics. On the contrary, its origin is clarified: it is the result of constant interactions and decoherence. Because of decoherence, the world experienced by humans is extremely stable and because of the frequent interactions, stable facts ascertained by different observers are in agreement. Hence, in practice, facts relative to one observer *do* place strict constraints on stable facts relative to another. This is why human creatures agree on the quantum state to assign to a system, on non-relational properties they assign to systems, and on the existence of a shared reality. RQM does not bring any subversion to the stability and coherence of this classical, macroscopic world. Instead, it shows that, by recognising the ultimately relative nature of events, we can have a coherent understanding of nature *beyond* the macroscopic regime in which the approximation that facts are *perfectly* stable is assumed to hold.

Another rhetorical move by Pienaar is to compare the RQM terminology with analogous terminology in different contexts. For instance, Pienaar writes

> Far from having de-mystified quantum mechanics by appealing to relations, RQM has merely mystified the concept of a 'relation'.

RQM takes the notions of physical system and quantum events happening between systems as primary. Quantum events involve two systems, are discrete, and are described by one variable of one system taking a value relative to the other system. The world is not described by the individual properties of individual systems, but by relative properties. These are called 'relations' because they involve more than one system. There is nothing mystifying in this terminology. 'Relations' have to be intended within this conceptual scheme, not in the conceptual scheme of classical mechanics, where they are subsidiaries of properties of individual systems.

All things considered, the main objection that Pienaar raises to RQM is not that it is inconsistent: it is that of being more radical than might appear at first sight.

9.3 On objectivity

This part of Pienaar's objections have to do with the consequences of **RQM:4** for notions of objectivity and the extent to which different perspective can be shown to agree.

No-go theorem 3

> **Dilemma:** RQM cannot consistently maintain both the principle of **RQM:6: shared facts**, and the principle of **RQM:4: comparisons are relative to one observer**. Rejecting one or the other leads to the following two horns:
> (i) If RQM rejects **RQM:6**, then it either implies *solipsism*, or else an *ontology of island universes* (these terms will be defined at the end of this section).
> (ii) If RQM instead rejects **RQM:4**, it becomes vulnerable to [no-go theorem 4].

As anticipated in section 9.1, Pienaar's formulation of **RQM:6**, is loose enough that it is either wrong, or a tautology. Let us see how the proof of this no-go theorem illustrates this point. Pienaar tries to derive the contradiction in the following way. Consider our two systems $\mathcal{F}$ and $\mathcal{W}$ interacting with $\mathcal{S}$. The quantum state of $\mathcal{S}$ relative to $\mathcal{W}$ or $\mathcal{F}$ will depend on the interactions between these three systems. He proceeds:

> Now suppose we have before us a description of $\mathcal{W}$'s account, and a description of $\mathcal{F}$'s account – laid out 'side by side' in a view from nowhere, so to speak – and we would like to know: *are these accounts mutually consistent?*

Pienaar correctly points out that

> according to **RQM:4**, this is not a well-posed question, because there is no 'view from nowhere'

and yet he also holds that

> **RQM:6** requires that this question be well-posed, for otherwise there would be no way to assert that two observer's accounts are in agreement.

If Pienaar intended **RQM:6** to imply that there is a 'view from nowhere,' from which to compare all accounts of reality, then clearly one must reject **RQM:6**, as it contradicts **RQM:4**. Crucially, however, there *is* a way to "assert that two observer's accounts are in agreement" (despite having rejected **RQM:6**): have $\mathcal{F}$ write down her account and let $\mathcal{W}$ read it and compare it with its own.[3] That $\mathcal{W}$ will find that $\mathcal{F}$'s account is in agreement with his, is precisely the content of **RQM:6**$\star$, which is clearly compatible with **RQM:4**.

Rejecting Pienaar's **RQM:6** and replacing it with **RQM:6**$\star$ amounts to grabbing horn (i) of the dilemma. Pienaar claims that this would plunge us into solipsism or into an ontology of island universes. Would it?

Solipsism?

The claim that RQM leads to "solipsism" has appeared elsewhere, especially in popular science (see for instance [113]).

In the philosophical literature and in common parlance, solipsism has nothing to do with incomplete of communication between physical systems. It is instead the idea that there is a single subject that exhausts all of reality and that the rest of reality only exists as the experience of that single subject.

This is exactly the opposite of RQM. The main assumption of RQM, its defining assumption, in fact, is the antithesis of solipsism: the world is not what is perceived by a single special entity—it is a network of interactions between equal status entities.

Pienaar does eventually concede[4] that probably RQM does not propose solipsism. He correctly characterises RQM's view: there are facts relative to every system,

[3] Or, in less anthropomorphised terms: let the dynamics be such that $\mathcal{F}$ encodes its account of $\mathcal{S}$ in a suitable pointer variable and let $\mathcal{W}$ interact with that variable.

[4] In fact, it is puzzling that he chooses to levy such a charge in the first place. He recently wrote an excellent comparison [206] between RQM and QBism, another interpretation often accused of being solipsistic. Neither interpretation is solipsist, for the same reason.

but that the different perspectives on reality, namely, the ensemble of facts relative to a single system, cannot be compared in an absolute manner; they can only be compared via a physical interaction. This is correct.

He calls this an ontology of "island universes." We do not find the name appropriate, let us see why.

Embodied knowledge

There is a subtle but important philosophical issue involved here. Consider the case in which the systems $\mathcal{F}$ and $\mathcal{W}$ are actually "observers" in the rich sense of the term. Say they are humans with laboratories, notebooks, and books, that store and process knowledge about the world. Let us focus on $\mathcal{F}$. What is the meaning of the statement that $\mathcal{F}$ has knowledge about the world, for instance about $\mathcal{S}$?

There are two possible answers. The first is a naturalistic answer. The second is a dualistic or idealistic answer. According to the first, this is a statement about the actual physical configuration of the ink and the notebooks, the charges in the computers and the synapses in the brain in $\mathcal{F}$, and about the correlations of these with whatever can be observed in $\mathcal{S}$. According to the second, $\mathcal{F}$'s knowledge is something over and above its physical configuration. In this case, the "inaccessibility" of $\mathcal{F}$'s knowledge, namely of the "universe as seen by $\mathcal{F}$," is indeed there. But this only follows because one assumes that knowledge is unphysical.

RQM adheres to a naturalistic philosophy. In a naturalistic philosophy, what $\mathcal{F}$ "knows" regards physical variables in $\mathcal{F}$. And *this* is accessible to $\mathcal{W}$. If knowledge is physical, it is accessible by other systems via physical interactions. It is precisely for this reason that knowledge is also subjected to the constraints and the physical accidents due to quantum theory. A physical interaction can and does destroy knowledge, because of standard Heisenberg uncertainty. Hence, ultimately, the intuition that disturbs Pienaar is a residual of anti-naturalism: the idea that knowledge can remain immune from quantum phenomena, because it can be disembodied.

Are relative facts needed?

Clarified this (subtle) point, there remains[5] in Pienaar's [205] an objection:

> this proliferation of disjoint universes is not motivated by observations, nor does it serve any explanatory purpose.

Every interpretation of quantum theory is "motivated by observations" in the sense that it is an attempt to devise a conceptual scheme that makes sense of a vast number of observations. More precisely, to make sense of the fact that observations are well described by quantum theory. As such, it is deeply rooted in observations: without observations, quantum theory—and its tentative interpretations—would never have appeared.

More to the point, what is the explanatory purpose of the multiplication of perspectives in RQM? The answer is that it offers a possible explanation to the key mystery of quantum physics: the apparent special role that "observers" seem to have in the theory. RQM illuminates this mystery by denying that there is anything

[5] Brukner reported a similar concern in an email [37].

special in observers, in the following general sense: facts happen relative to any system (**RQM:1**). What *is* special in a (large class) of macroscopic observers is only that decoherence and frequent interactions stabilise and render consistent for them many relative facts. RQM is the observation that quantum physics can be made sense of also beyond the limit of perfect decoherence.

Thus, the "explanatory purpose" of RQM's multiplication of perspectives (the idea that facts happen at interactions between any two systems) is that it serves as a possible solution to the measurement problem. It helps to answer questions like:

- Q: When does something become a fact?
  A: Something becomes a fact, relative to you, when you interact with a system.

- Q: How does Schrödinger's cat feel?
  A: Either awake looking at the vial, or asleep having a dream. The cat does not stop having experiences only because the box is sealed off from the rest of the lab.

- Q: What physical systems are measuring apparata?
  A: Any system whose pointer variables (i) get appropriately entangled to a variable you are interested and (ii) with which you can interact.

- Q: When does the wavefunction collapse?
  A: The wavefunction for $\mathcal{S}$ relative to $\mathcal{W}$ collapses whenever $\mathcal{W}$ interacts with—gets a kick from—$\mathcal{S}$ and therefore $\mathcal{W}$ gathers information about $\mathcal{S}$.

Island universes

The expression "island universes" that Pienaar uses to RQM's discredit is taken from Huxley's *The Doors of Perception* [141], where "island universes" is applied to conscious experiences. The situation with conscious experiences is in fact analogous to that in quantum physics, but instead of weakening the motivation for postulating multiple perspectives, it strengthens it.

Let's see. Do we have direct evidence that other humans have a first-person experience of reality like ours? We do not. Do we hold that thinking of other humans as having experiences like ours is a hypothesis that is "not motivated by observations, nor [serves] any explanatory purpose"? Of course not! The alternative is to think that we ourselves are the only conscious being in the universe. This is solipsism! We have ample reasons to believe that we share conscious experiences with (at least) other humans. By the same token, RQM points out that we have reasons to believe that we share the reality of perspectival facts with any physical system.

This is the core of RQM: we understand that we are normal physical systems and, as such, we are affected by the rest of reality. Hence we make a reasonable extrapolation, based on this and on our realisation that we are not special. We have no reason to believe that reality comes into being only when it interacts with us, and not also when anything interacts with anything else. That there is no fundamentally distinguished class of systems called "the classical world" or "measuring apparata" that have the privileged ability of actualising the variables of other systems.

Finally, Pienaar complains that the different "views" do not "share facts". Here, Pienaar puts undue restrictions on what is a shared fact. The analogy with conscious experience helps us here, too. Can two people "share" the same experience? It depends what we mean by that. If we mean to ask if two people can have the exact same set of sensory experiences at the same time and think the exact same thoughts, then clearly no. But this is not how we normally understand the phrase. We share experiences when we listen to the same performance of an orchestra, when we watch a movie together, when we analyse the same object together. And we can verify that we are sharing experiences by comparing our mental lives—not in some sort of absolute external sense, but by interacting with (talking and listening to) each other. The two internal mental lives are still different after talking, but the two people can nevertheless reach an intersubjective agreement.

In the ontology of RQM, two systems $\mathcal{F}$ and $\mathcal{W}$ cannot share the same facts about a third system $\mathcal{S}$ in the sense that whenever there is a quantum event for $\mathcal{F}$, there is also immediately a quantum event for $\mathcal{W}$. It is not even the case that a later interaction between $\mathcal{W}$ and $\mathcal{F}$ can make a previous quantum event between $\mathcal{S}$ and $\mathcal{F}$ a quantum event for $\mathcal{W}$. What they *can* do, however, is verify that there is a consistency between their shared perspective, by interacting with (or measuring, as Pienaar puts it) each other. In this sense, $\mathcal{F}$ and $\mathcal{W}$ end up sharing facts: the behaviour of $\mathcal{F}$ and $\mathcal{S}$ that $\mathcal{W}$ observes is coherent with the assumption that $\mathcal{F}$ sees the same $\mathcal{S}$ as $\mathcal{W}$ does.

The fear that this destroys the coherence of the world or throws us into a solipsistic nightmare is similar to the fear that by setting the Earth in motion Copernicus challenged the stability of the houses built on Earth. Yet, quantum physics teaches us that $\mathcal{W}$ could also interact with $\mathcal{F}$ in a way that destroys $\mathcal{F}$ account of their previous interaction with $\mathcal{S}$. Is this surprising? Perhaps, but this is what quantum physics implies.

The loose frame loophole

Pienaar also raises a concern regarding the ambiguous way in which some RQM literature talks about facts relative to different systems. This is a valid concern. It has already been echoed out by at least one other source [248]. This ambiguity is a defect of the original literature.

Statements such as "when two systems $\mathcal{F}$ and $\mathcal{W}$ interact with a system $\mathcal{S}$, the perspectives of $\mathcal{W}$ and $\mathcal{F}$ agree, and this can be checked in a physical interaction", which can be found in the RQM literature, mean *only* that $\mathcal{W}$ can interact with $\mathcal{F}$'s pointers and check that they were affected in its interactions with $\mathcal{S}$ in a way consistent with what $\mathcal{W}$ directly learns about $\mathcal{S}$. This is the content of **RQM:6**$\star$ again. Obviously, $\mathcal{F}$ can do the same with $\mathcal{W}$ (**RQM:1**).

This is normally left implicit whenever one talks about facts relative to different observers, assuming that it is clear enough to fill in the gaps. This is sometimes easier and sometimes harder. Indeed, Pienaar brings up [85] (from which chapter 8 was adapted) as an example in which things are more complex than even the authors of the original paper realised. Let us look at this example in detail and make sure that the loophole is closed, as this can serve as an example for other situations.

The central point of 8 and [85], is the definition of a *stable fact.* A fact relative to $\mathcal{F}$ is said to be *stable for* $\mathcal{W}$ if classical probability calculus can be used to compute the probability of an event for $\mathcal{W}$ using this fact relative to $\mathcal{F}$. More specifically, assume that, from $\mathcal{W}$'s perspective, $\mathcal{F}$ interacts with a variable A of $\mathcal{S}$; According to **RQM:1**$\star$ and **RQM:4**$\star$, this interaction may result in the value of A to become a fact relative to $\mathcal{F}$, but no fact is established relative to $\mathcal{W}$. However, the value of A relative to $\mathcal{F}$ is considered *stable for* $\mathcal{W}$ if, in computing the probability for a variable B to taking the value b relative to $\mathcal{W}$ in a subsequent interaction, we can write:

$$P(b^{(\mathcal{W})}) = \sum_i P(b|a_i)P(a_i^{(\mathcal{F})}). \tag{9.6}$$

In the formula above, we have inserted superscripts to highlight that stability allows to mix perspectives. At an operational level, it allows to reason *as if* there is epistemic uncertainty about the value of A relative to $\mathcal{W}$, even though, ontologically, A does not have an actual value relative to $\mathcal{W}$. The conditional probability

$$P(b|a_i) = |\langle b|a_i\rangle|^2 \tag{9.7}$$

does not need[6] superscripts: the transition probabilities are the main outputs of quantum theory and they define the probability of facts relative to a system, given other facts relative to the *same* system.

Pienaar expresses his doubts that a formula like (9.6) can ever make sense in RQM, as it relates probabilities of facts relative to *different* systems. Indeed, while quantum theory allows the computation of the transition probabilities $P(b|a_i)$ as well as the probability $P(b^{(\mathcal{W})})$ (given the state of $\mathcal{S}-\mathcal{F}$ relative to $\mathcal{W}$), the quantities $P(a_i^{(\mathcal{F})})$ do not have meaning in RQM, *a priori*. But this is precisely the point of the definition. The $P(a_i^{(\mathcal{F})})$ *acquire* this meaning when the relation between $P(b^{(\mathcal{W})})$ and $P(b|a_i)$ is given by (9.6). In other words, when the interaction between $\mathcal{S}$ and $\mathcal{F}$ (as described by $\mathcal{W}$) is such that (9.6) holds for *some* probability distribution $P(a_i^{(\mathcal{F})})$, then $P(a_i^{(\mathcal{F})})$ acquires the meaning of a probability distribution over the possible values of A—even though the value of A is not a fact relative to $\mathcal{W}$. The value of A might be a fact relative to $\mathcal{F}$, hence the superscript. We invite the reader to revisit section 8.1.2 as a concrete example.

As Pienaar remarks, the de-labelling is only methodological. Even when (9.6) holds, there is no ontological identification of a fact relative to $\mathcal{F}$ with a fact relative to $\mathcal{W}$. For all practical purposes, different systems in the same stability class act *as if* they live in a macroreality of absolute facts and *as if* they share facts.

No-go theorem 4

This theorem is the second horn of the dilemma that no-go theorem 3 was supposed to offer. We grabbed the first horn, so we are not required to answer to this, but we will do anyway, because is another example of the mischaracterisation of the role of the quantum state. The theorem is in a form of a trilemma:

> **Trilemma:** The propositions **P1** & **P2** and the claim **RQM:3** cannot all be true.

[6] In [85], we wrote $P(b^{(\mathcal{W})}|a_i^{(\mathcal{F})})$, but, as Pienaar remarks, this notation is highly misleading; a better notation would have been $P(b^{(\mathcal{W})}|a_i^{(\mathcal{W})})$. Better still to omit the superscript as we have done in chapter 8, since the transition probabilities given by the Born rule (9.7) are independent of the context $\mathcal{W}$.

where

> **P1.** $\mathcal{W}$ can measure $\mathcal{F}$-$\mathcal{S}$ in any basis at Event 2, independently of which basis $\mathcal{F}$ measured $\mathcal{S}$ at Event 1.

and

> **P2:** Suppose $\mathcal{W}$ measures $\mathcal{F}$-$\mathcal{S}$ in the $\{|Fy_m\rangle\,|y_n\rangle\}$ basis and obtains some outcome, updating the state relative to $\mathcal{W}$ to one of the states in $\{|Fy_n\rangle\,|y_n\rangle\}$ just after Event 2. Then we can interpret this state as indicating that '$\mathcal{F}$ measured $\mathcal{S}$ in the $\{|y_n\rangle\}$ basis and obtained one of the outcomes in the set $\{y_n\}$ at Event 1'.

The solution is simple: **P1** and **RQM:3** are true in RQM, while **P2** is false. This is again caused by the wrong formulation of **RQM:5**. The fact that $\mathcal{S}-\mathcal{F}$ is in the state $|y_n\rangle_{\mathcal{S}}\,|Fy_n\rangle_{\mathcal{F}}$ does not imply that the value of Y is a fact for $\mathcal{F}$. Indeed, one way to prepare such a state is to start with $\mathcal{S}$ and $\mathcal{F}$ uncorrelated and just rotate each system separately into $|y_n\rangle_{\mathcal{S}}\,|Fy_n\rangle_{\mathcal{F}}$. In this case $\mathcal{S}$ and $\mathcal{F}$ never interacted and there could not be a fact about $\mathcal{S}$ relative to $\mathcal{F}$. Or, in the operational language, $\mathcal{F}$ did not measure $\mathcal{S}$.

No-go theorem 5

This last no-go theorem again fails because of Pienaar's wrong formulation of **RQM:5**. The theorem considers particular states of the $\mathcal{S}-\mathcal{F}$ system and tries to derive something about facts of $\mathcal{S}$ relative to $\mathcal{F}$ from these states. Again, this is not a possible logic in RQM. The states in question (as Pienaar himself points out) are states relative to $\mathcal{W}$. What they contain is information about what $\mathcal{W}$ can measure, namely how the $\mathcal{S}-\mathcal{F}$ system has affected $\mathcal{W}$ or can affect $\mathcal{W}$ is the future. Trying to read out from these states the full facts relative to $\mathcal{F}$ is is not something compatible with RQM.

9.4 Qubits are not observers

Let us now come to Brukner's no-go theorem [38]. Like Pienaar's results, this is a correct mathematical observation that, instead of providing a criticism of RQM, serves to sharpen the interpretation. His explicit aim is presented in the introduction:

> I will derive a no-go theorem that restricts the possibility of understanding the relational description in RQM as knowledge that one system can have about another in the conventional sense of that term.

Part of what makes Brukner's result seem a challenge towards RQM, is Brukner's use of operational language (such as "measurement," "observer," and "knowledge") to formulate his no-go theorem even though, as he himself remarks, "RQM makes not reference to [these] concepts". The other aspect that contributes to the confusion is his overplaying the role of the quantum state. Like in Pienaar's no-go theorems 1, 2, 4, and 5, Brukner tries to read relative facts between two systems by looking at the state assigned to these by a third system, while RQM does not allow this.

The setup of the theorem is essentially that of Pienaar's no-go theorem 1. Two systems[7] S and O are in some potentially entangled state $|\psi\rangle_{SO}$. Note that here O stands for "observer"; there is no restriction on the nature of S and O (they can be qubits); and Brukner does not specify what the state $|\psi\rangle_{SO}$ is relative to, we take it as relative to a third system W. The theorem states that the two following things cannot both be true.

> **1. (DefRS) Definite Relative State** For *any* set of states $\{|x_i\rangle_S, |X_i\rangle_O\}$ such that
>
> $$|\psi\rangle_{SO} = \sum_i c_i |x_i\rangle_S |X_i\rangle_O , \tag{9.8}$$
>
> [...] the states $|X_i\rangle_O$ are states of knowledge of the observer. When the observer is in state $|X_i\rangle_O$ she knows that the system is in the definite relative state $|x_i\rangle_S$.
>
> **2. (DisRS) Distinct Relative State** The observer's states of knowledge $|X\rangle_O$ and $|X\rangle_O$, which are correlated with distinct relative states $|x\rangle_S$ and $|x'\rangle_S$ of the system, are represented by orthogonal vectors in the observer's Hilbert space, i.e. if $|x\rangle_S \neq |x'\rangle_S$, then $\langle X|X'\rangle = 0$.

The proof of the theorem is pretty straightforward, as it also employs that the Schmidt decomposition of an entangled state need not be unique.

Since RQM makes no appeal to a notion of knowledge, it is not clear why this should be a challenge to RQM. From RQM's perspective, Brukner's result ostensibly is a no-go theorem about the meanings that the word "knowledge" can assume, given what we know about quantum mechanics. Indeed, we see two ways of reading this result, either

- **DefRS** is taken as a *definition* of the word "knowledge," and then **DisRS** is false, or
- **DisRS** is a constraint on what can be a "state of knowledge", and then **DefRS** is false.

Le us consider these two options in turn.

If we take **DefRS** to define knowledge, then O has knowledge about S in the sense that W can learn about the probabilities of future interaction with S by interacting with O. This is the same well-defined sense in which a given set of pixels on a computer screen have knowledge about the time and a given set of ink molecules in a book have knowledge about lasers: by interacting with those molecules we expect to learn about future interactions with coherent light. In this sense, "knowledge" is nothing more—and nothing less—than correlations between two systems, as expected by a third. Then the failure of **DisRS** tells us nothing we didn't know already: when S and O are entangled, interacting with different variables of S affects our information about different variables of O.

Consider now using **DisRS** as a constraint on what is a state of knowledge. Then the failure of **DefRS** implies that one has to have correlations on a preferred basis before talking about knowledge. This is closer to the other meaning of the

[7] We switch from $\mathcal{S}$ and $\mathcal{F}$ to S and O to be closer to Brukner's [38].

word “knowledge” as applied to complex systems such as agents and conscious observers. This is perhaps “the conventional sense” that Brukner has in mind in the introduction. Note that this is still a naturalistic use of the world, as it refers to the physical properties of such observer. Then a superposition of two states of knowledge is not a new state of knowledge, but a superposition of two states of knowledge.

Both choices are valid. The only problem is confusing the different possible meanings of the word knowledge. And that is why the RQM literature warns against using terms that are normally reserved for macroscopic physics when talking about the fundamental elements of the theory.

In sum, we agree with Brukner that “qubits are not observers,” for the uncontroversial fact that qubits are not decision-making agents capable of processing information. That has never been a claim of RQM. The controversial claim that RQM makes is **RQM1⋆ (facts happen relative to any physical system)**. Brukner's no-go theorem has no impact on this, since a state such as (9.8) that W assigns to S and O is not enough for W to infer what might or might not be a fact for O, as explained in detail in section 9.2 when discussing Pienaar's no-go theorem 1.

9.5 Conclusion

Pienaar's [205] presents arguments against two ideas: that (i) RQM preserves certain classical relativistic intuitions about relations and that (ii) it preserves the idea that consistency can be established between different observers' accounts. Both conclusions are correct: (i) RQM does not *preserve* certain classical relativistic intuitions about relations: it *extends them* and makes them more radical (“facts are relative”). And, (ii) RQM does not preserve the idea that consistency can be established between different observers' accounts. It replaces it with the idea that systems communicate in the sense that they can measure (quantum mechanically!) each other's pointer variables. Since I myself am an observer, I find nothing strange in the idea that you could read pointer variables in me that get correlated with external variables when these are realised with respect to me. Brukner's [38] argues that if we want to call the entanglement of two systems “knowledge” that the two systems have about one another, then this “knowledge” differs in some radical way from common usage.

These objections do not challenge the coherence of RQM. They maybe show that RQM is more radical than what it might appear at a first sight.

Does an ontology where views cannot be compared directly and physical systems can only check each other via quantum measurements imply solipsism? No, it does not. Does it change in depth our way of thinking about reality? Yes it does. Quantum mechanics is radical. One way or the other, we have to embrace it, not try to tame it.

Conclusion

The meeting of quantum gravity with quantum information theory and quantum foundations research is an exciting development in fundamental physics. The work reported in this thesis covers a fraction of the possibilities and themes of this new interdisciplinary field.

We have considered in detail one of the opportunities it created: the exciting prospect of empirically testing a quantum gravity prediction—superpositions of geometry—via the detection of gravity mediated entanglement. This will be a landmark experiment that will rule out either all classical field theories of gravity or quantum gravity itself. In chapter 4, we have seen an example of how quantum computation and quantum optics can help illuminate the claims of this experiment, and prepare for its practical applications. In chapter 5, we used standard quantum field theory techniques to see how the quantum information claim that observing gravitationally mediated entanglement certifies non-classicality of the gravitational field applies to linearised quantum gravity, our best theory of quantum gravity in this regime. We have also derived formulas which can be used to quantitatively test this theory using this experiment. In chapter 6, we saw how a similar experimental setup would allow us to probe time intervals at unexplored regimes, and try to observe planckian discreteness of time by observing discontinuities in the phase of an interferometer in weak gravitational fields.

An exciting prospect for low energy exploration of quantum gravity in the lab is the recreation of geometrical-like properties through the engineering of entanglement. Several authors, have argued on different theoretical grounds for the existence of a deep link between geometry and entanglement [27, 41, 131, 142, 162]. Advances in quantum control of matter might soon allow to explore this connection in the lab using low energy quantum systems [203]. Connecting the approaches of geometry from entanglement and quantum superpositions of geometry, both experimentally and theoretically, would be an exciting research direction.

On the theoretical side, recent developments in the subject of quantum reference frames [14, 116, 257] represent another interesting confluence in the methods and research of quantum gravity and quantum information theory. It has lead to a unification of three major approaches to resolving the problem of time in quantum gravity [129]. This work on quantum reference frames relates the concepts of gauge redundancy with the choice of reference frame. It allows to "switch perspectives," change the quantum system that acts as a reference frame. A striking aspect from the point of view of this thesis: switching perspective does not entail collapsing the wavefunction, hence this approach seems more natural from an Everettian interpretation of the wave function. It would be interesting to study how this formalism describes the Wigner friend scenario, and how it relates to the idea of relative facts.

But there are also tensions in this interdisciplinary convergence. The communities of quantum gravity, on one side, and quantum information and foundations on the other, tend to have a different regard for the status of operationalism, agency, and causality. In chapter 7, we argued that the time-orientation of the operational formalism can be understood as the time-orientation of agents and laboratories. Two related challenges follow from this observation. The first, mathematical: how to adapt the operational formalisms to a time-reversal symmetric formalism [82]. To this aim, the formalisms introduced in [126] and [235] seem to be important strides in this direction. The second, harder: to derive a convincing model of the connection between the thermodynamic arrow of time and the time-orientation of agency. At the moment, the relation between the thermodynamic arrow of time and the arrow of causation is not known.

Looming above all, the interpretation of quantum mechanics. The approach used in practice, the no-interpretation approach, is well-suited for laboratory physics but it is of dubious application for fundamental physics. Arguably, if agency is not a fundamental aspect of the physical world but emergent from microphysics, concepts of observers and measurement apparata are out of place in fundamental formulations of physics. The relational interpretation of quantum mechanics was developed without observers and measurements as primitives, with the problem of quantum gravity in mind [221]. This interpretation is based on the concept of relativity of facts. Surprisingly enough, we are starting to have experimental evidence for the relativity of facts [32, 212]. Results from experimental metaphysics [32] show that local, non-superdeterministic theories cannot explain certain experimental phenomena without invoking the relativity of facts. In chapters 8 and 9, we fleshed out some of the consequences of the relativity of facts from the point of view of relational quantum mechanics. We saw how the classical world emerges from relative facts and to what extent different systems inhabit the same world. However, relative facts force us to re-think our notions of locality [169, 204] and of events happening in spacetime [50], and much work remains to be done.

I hope this thesis manages to convey a fraction of the beauty, possibility, and mystery that stem from exploring space and time with the aid of low energy quantum systems, a field which opens a new door for the empirical exploration fundamental physics.

Appendix

Appendix A

Works related to chapter 7

The exploration of the time-symmetry of quantum uncertainty started early with Einstein, Tolman, and Podolski [95] already noting in 1931 that "the principles of quantum mechanics actually involve an uncertainty in the description of past events which is analogous to the uncertainty in the prediction of future events."

Aharonov, Bergmann, and Lebowitz [4] built a time-symmetric theory out of quantum theory by considering the frequencies of outcomes of a sequence of projective measurements in ensembles constructed by pre- and post-selection. They note that this theory is time-symmetric in the sense that the frequencies observed in one ensemble are the same as the ones observed in the ensemble prepared by swapping the pre- and post-selection and performing the measurements in the reversed order. They note that probabilities calculated based only on preselection are experimentally accurate, while probabilities calculated only on post-selection are insufficient. They argue that this time-reversal asymmetry is not inherent to quantum mechanics but is a consequence of the asymmetry of the macroscopic world. Finally they advance that this asymmetry should be represented by adding a time-asymmetric postulate to the time-symmetric theory. This postulate is the precursor of the "no signalling from the future" axiom. In light of the argument in this chapter, there is no need to build a time-symmetric theory and then break the symmetry. Quantum mechanics is already symmetric in the sense that generally it does not distinguish prediction and postdiction, or the past from the future. Time-reversal asymmetry is a feature of the operational formulations of quantum theory.

While Aharonov *et. al.* [4] considered a sequence of measurements "sandwiched" between selection events, the setup in this chapter is similar to that in [259], where Watanabe was concerned with calculating the probabilities for a past or future event given a present event. Watanabe introduced *retrodictive quantum mechanics*, where a state is assigned to the system based on present data and then evolved to past times. Like Aharonov *et. al.*, Watanabe remarks that "blind retrodiction," (*i.e.* what we call prediction in this chapter: retrodiction with flat priors on the past events) does not work well in practice because agents in the past can decide to interrupt or go on with the experiments. Watanabe also recognises that inference is inherently asymmetric and can run in either direction of time, starting from data at a given instant.

Retrodictive quantum mechanics has been further developed over the decades by Barnett, Pegg, Jeffers and collaborators [12, 198, 247], who devised general postdiction formulas, including ones equivalent those we derive in sections 7.2 and 7.4.2 for postdiction in closed systems and for quantum channels respectively. Our contribution to retrodictive quantum mechanics is twofold. First, we propose the explanation of the asymmetry between prediction and postdiction in quantum channels in terms of implicit data about the past of a purifying system. Secondly, we prove *no-signalling from the unknown*, which is a property of quantum mechanics, and in particular of retrodictive quantum mechanics, that to our knowledge has not been recognised before. We also pay attention to the conceptual difference between retrodiction and time-reversal, and we relate these two concepts.

Leifer and Pusey [157] consider a similar prepare-and-measure scenario as us and investigate what time-symmetry can imply on possible ontological models for quantum theory. Their definition of an *operational time reverse* formally equivalent to what we call an active time-reversal. They define *operational time-symmetry* as the existence of an operational time reverse. They derive a fascinating no-go theorem for ontic extensions (aka hidden variable models) of operational time-symmetric processes. We share the belief that operational time-symmetry is an essential feature of quantum mechanics, but we do not concern ourselves with ontic extensions. Instead, we are interested in understanding why not all quantum channels are operationally time-symmetric. We also study the difference and relation between postdiction and time-reversal, prove that operational time-symmetry is equivalent to inference symmetry.

Oreshkov and Cerf [192] define an extension to operational quantum theory, allowing for a "notion of operation that permits realisations via both pre- and post-selection." Their motivation for building a new theory is stated in the abstract: "The symmetry of quantum theory under time reversal has long been a subject of controversy because the transition probabilities given by Born's rule do not apply backward in time." Our work shows that the Born rule applies equally well in both directions in time—as long as we treat prediction and postdiction on equal footing. We argue that the asymmetry of operational quantum theory reflects the asymmetry of the agents, not the asymmetry of quantum phenomena *per se*.

The work in this chapter was inspired by conversations at the QISS conference at HKU and at the QISS virtual seminars [188], where it transpired that the time-asymmetric operational formulation obfuscated the fundamental time-symmetry of quantum theory.

The importance of separating the physical from the inferential in quantum mechanics is a more modern idea, perhaps traceable to E.T. Jaynes who famously compared quantum theory to an omelette to be unscrambled [145]. The QBists see quantum theory as not much more than the correct probability calculus to use in our world [108]. Caticha has often emphasised the importance of understanding probability first and foremost as an inferential tool [46]. Leifer and Spekkens [158] have formally developed the analogy between quantum probabilities and Bayesian inference. They introduce a notion of "quantum conditional states" representing sequential and parallel quantum experiments and prediction and postdiction on the same footing, just like in classical probability theory. In our work, we limit ourselves to classical probability theory. At first, we use the Born rule to obtain classical conditional probability distributions $P_{pre}(x|a, U)$ for prediction probabilities in sequential experiments. We show that the Born rule actually can be used to compute prediction and postdiction probabilities.

Appendix B

Computation in EM

The action for electromagnetism coupled to a four current j_μ is of the form $S = S_\mathrm{M} + S_A$ with

$$S_A = \int \mathrm{d}^4x \left(-\frac{c^2}{16\pi k_e} F_{\mu\nu}F^{\mu\nu} + j_\mu A^\mu \right). \tag{B.1}$$

Here, $d^4x = dt d^3\boldsymbol{x}$, S_M is the free matter action that also includes the B_0 coupling to the spins, A^μ is the four-potential, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the field strength and k_e Coulomb's constant. We use greek indexed latin letters for 4–vectors and bold latin letters for 3–vectors. The metric signature is $(-,+,+,+)$ and $\eta_{\mu\nu}$ is the Minkowski metric.

The action S_A is gauge invariant. We will express the Lagrangian in the Lorentz gauge $\partial_\mu A^\mu = 0$ to simplify calculations. Boundary terms at infinity are taken to vanish. Integrating by parts, the action then reduces to

$$S_A = \int \mathrm{d}^4x \left(-\frac{c^2}{8\pi k_e} \partial_\mu A_\nu \partial^\mu A^\nu + j_\mu A^\mu \right), \tag{B.2}$$

and the equations of motion are

$$\Box A^\mu = -\frac{4\pi k_e}{c^2} j^\mu, \tag{B.3}$$

where $\Box = \partial_\mu \partial^\nu$.

Now, we obtain the on–shell action by placing (B.3) into (B.2) and integrating again by parts to obtain

$$S_A^\mathrm{os} = \frac{1}{2} \int \mathrm{d}^4x \; j_\mu A^\mu. \tag{B.4}$$

The entire contribution of the electromagnetic field to the on–shell action is encoded in this expression. As we saw in the main text, S_A^os is the central object of interest for induced entanglement: this Lorentz covariant and gauge invariant quantity is the observable that would be measured in an experiment aiming to observe induced entanglement.

We now consider the electromagnetic interaction of two point charges. The four-current is given by

$$j^\mu(x) = \sum_a q_a v_a^\mu(t) \delta^{(3)}(\boldsymbol{x} - \boldsymbol{x}_a(t)), \tag{B.5}$$

where $v_a^\mu(t) = (c, d\boldsymbol{x}_a/dt) = (c, \boldsymbol{v}_a)$. From the equations of motion, assuming no radiation incoming from past infinity, the potential of this charge configuration is given in the Lorentz gauge by the well-known Liénard–Wiechert potentials [121]

$$A^\mu(t, \boldsymbol{x}) = \frac{k_e}{c^2} \sum_a \left[\frac{q_a v_a^\mu}{d_a - \boldsymbol{d}_a \cdot \boldsymbol{v}_a / c} \right]_{t=t_a} \tag{B.6}$$

with $\boldsymbol{d_a}(t) = \boldsymbol{x} - \boldsymbol{x}_a(t)$, $d_a = |\boldsymbol{d_a}|$. Crucially, all quantities on the right hand side are evaluated at the *retarded* time t_a, defined implicitly as the solution of

$$ct_a = ct - d_a(t_a). \tag{B.7}$$

The retarded time t_a is the time t at which the past lightcone of $(t, \boldsymbol{x})$ intersects the (timelike) trajectory of particle a.

By placing (B.6) and (B.5) in (B.4) and performing the space integration, we get an explicit expression for the on–shell action giving the interaction between the two charges

$$S_A^{\text{os}} = \frac{k_e}{2c^2} \sum_{a,b}^{a \neq b} \int \mathrm{d}t \, \frac{q_a \, q_b \, v_a^\mu(t_{ab}) \, v_{b\mu}(t)}{d_{ab}(t) - \boldsymbol{d}_{ab}(t) \cdot \boldsymbol{v}_a(t_{ab})/c}. \tag{B.8}$$

Here, the retarded time t_{ab} is defined as the implicit solution of

$$ct - ct_{ab} = |\boldsymbol{x}_b(t) - \boldsymbol{x}_a(t_{ab})| \tag{B.9}$$

and we also defined

$$\boldsymbol{d}_{ab}(t) = \boldsymbol{x}_b(t) - \boldsymbol{x}_a(t_{ab}) \tag{B.10}$$

and $d_{ab} = \boldsymbol{d}_{ab}$.

In the slow moving approximation $|\boldsymbol{v}_a| \ll c$, the exact expression (B.8) approximates to

$$S_{\mathcal{F}}^{os} = -\frac{1}{2} k_e \sum_{a,b}^{a \neq b} \int \mathrm{d}t \frac{q_a q_b}{d_{ab}(t_{ab}, t)}. \tag{B.11}$$

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