

Title: Can we think time-symmetrically about causation?

Speakers: Andrea Di Biagio

Series: Quantum Foundations

Date: December 10, 2020 - 10:00 AM

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Abstract: We often say that quantum mechanics allows to calculate the probability of future events. In fact, quantum mechanics does not discriminate between predicting the future or postdicting the past. I will present the results of a recent work by Rovelli, DonÃ and me, where we address the apparent tension between the time symmetry of elementary quantum mechanics and the intrinsic time orientation of the formulations of quantum theory used in the quantum information and foundations communities. Additionally, I will sketch a way to think time symmetrically about causality in quantum theory by using the new notion of a causal-inferential theory recently proposed by Schimd, Selby and Spekkens.



SAPIENZA  
UNIVERSITÀ DI ROMA

# Can we think timelessly about causation?

Andrea Di Biagio

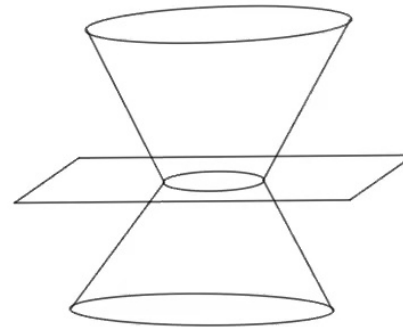
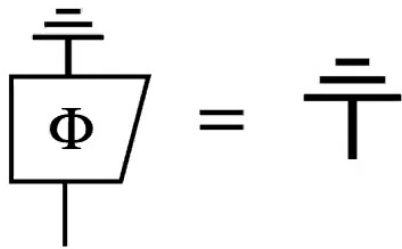
10 Dec 2020

Perimeter Institute

Quantum Foundations Seminar

## Starting tension

**No signalling from the future:** An OPT is **causal** if the probabilities of an operation do not depend on the choice of any *later* operation.



**Relativistic Causality:** A change in the initial data in a region  $S$ , does not produce any change in the regions outside the causal *past and future* of  $S$ .

# Reconstructions

- Lucien Hardy, “Quantum Theory From Five Reasonable Axioms,” (2001), [arXiv:quant-ph/0101012](#).
- Borivoje Dakic and Časlav Brukner, “Quantum theory and beyond: Is entanglement special?” (2009), [arXiv:0911.0695 \[quant-ph\]](#).
- Lluís Masanes and Markus P. Müller, “A derivation of quantum theory from physical requirements,” *New Journal of Physics* **13**, 063001 (2011).
- G. Chiribella, G. M. D’Ariano, and P. Perinotti, “Informational derivation of Quantum Theory,” *Physical Review A* **84**, 012311 (2011), [arXiv:1011.6451](#).
- Lucien Hardy, “Reconstructing quantum theory,” (2013), [arXiv:1303.1538 \[gr-qc, physics:hep-th, physics:quant-ph\]](#).
- Philipp A. Höhn, “Toolbox for reconstructing quantum theory from rules on information acquisition,” *Quantum* **1**, 38 (2017), [arXiv:1412.8323](#).
- Philipp A. Höhn and Christopher Wever, “Quantum theory from questions,” *Physical Review A* **95**, 012102 (2017), [arXiv:1511.01130](#).
- John H. Selby, Carlo Maria Scandolo, and Bob Coecke, “Reconstructing quantum theory from diagrammatic postulates,” [arXiv:1802.00367 \[quant-ph\]](#) (2018), [arXiv:1802.00367 \[quant-ph\]](#).
- Ding Jia, “Quantum from principles without assuming definite causal structure,” *Physical Review A* **98**, 032112 (2018), [arXiv:1808.00898](#).
- Robert Oeckl, “A local and operational framework for the foundations of physics,” *Advances in Theoretical and Mathematical Physics* **23**, 437–592 (2019), [arXiv:1610.09052](#).

# Reconstructions

Does quantum uncertainty imply time orientation?

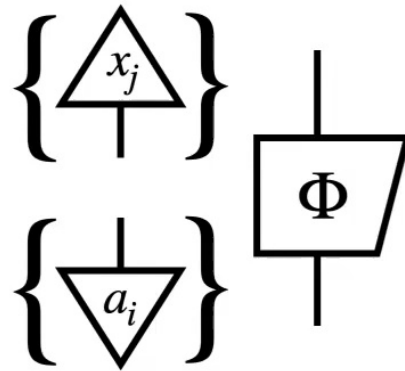
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# Plan

- **Quantum Information and the arrow of time**
- **Towards time-symmetric causation**

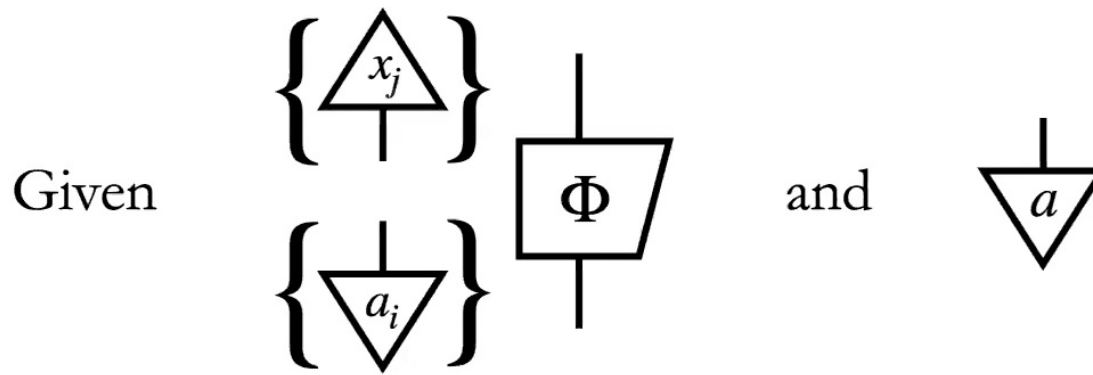


## Two Games



## Prediction vs Postdiction

**Prediction:** Given a preparation, a test and the result of the preparation, calculate the probabilities of the outcomes of the test.

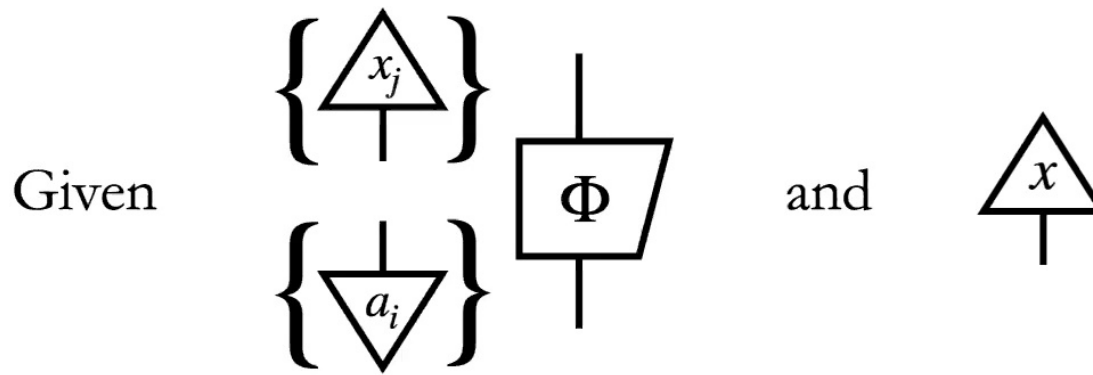


find  $P_{pre}(x_j | a, \Phi)$



## Prediction vs Postdiction

**Postdiction:** Given a preparation, a test and the result of the *test*, calculate the probabilities of the outcomes of the *preparation*.



find  $P_{post}(a_i | x, \Phi)$



## Closed Systems

We are doing inference using the Born rule.

$P(a)$  and  $P(x)$  are *a priori* probabilities.

**Prior**  $P(a) = \frac{1}{d}$

**Data**  $P(x) = \sum_{i=1}^d P_{pre}(x | a_i, U)P(a_i) = \sum_{i=1}^d |\langle x | U | a_i \rangle|^2 \cdot \frac{1}{d} = \frac{1}{d}$

$$P_{post}(a | x, \Phi) = |\langle x | U | a \rangle|^2 = P_{pre}(x | a, U)$$

## Time agnostic probabilities

A process  $\Phi$  is **inference symmetric** if:

$$P_{pre}(x_j | a_i, \Phi) = P_{post}(a_i | x_j, \Phi)$$

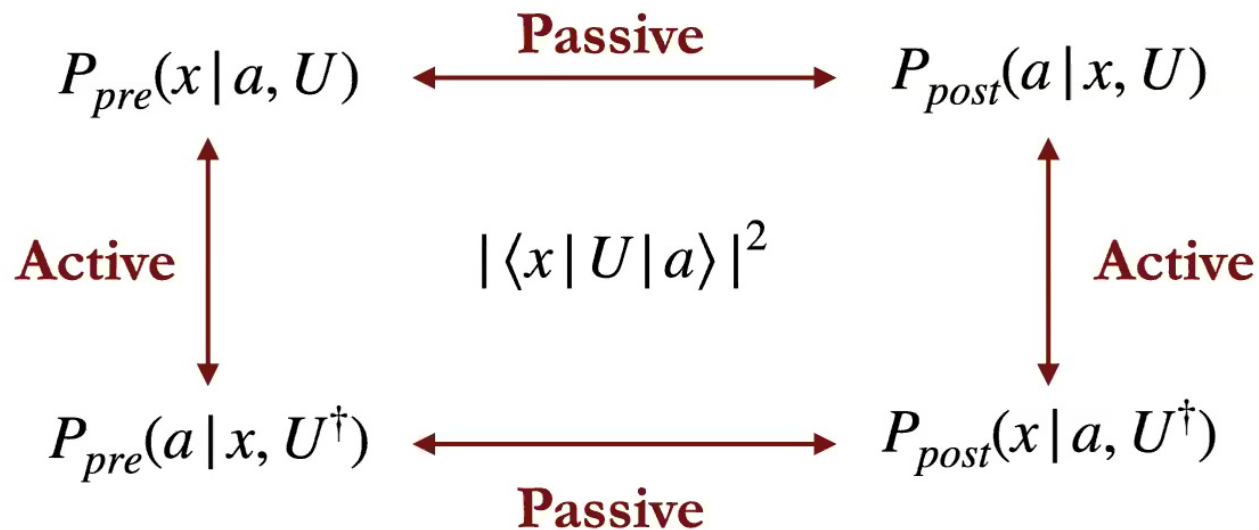
for any choice of bases.

Closed quantum systems are inference symmetric.

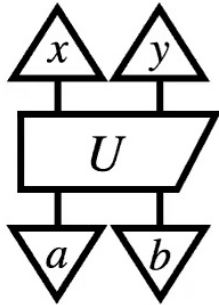
## Time-Reversal

**Passive:** Describe physical events in reversed order.

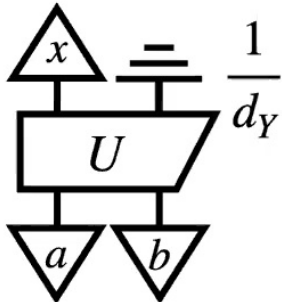
**Active:** Find a process that undoes the original process.



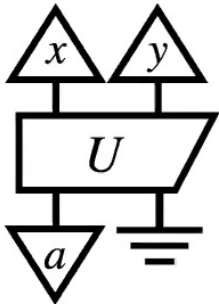
# Open Systems



$$P_{pre}(xy | ab, U) = P_{post}(ab | xy, U)$$



$$P_{post}(ab | x, U) = \frac{1}{d_Y} \sum_{i=1}^{d_Y} P_{post}(ab | xy_i, U)$$



$$P_{post}(a | xy, U) = \sum_{i=1}^{d_B} P_{post}(ab_i | xy, U)$$

## Direction of inference

$$P_{pre}(xy | a, U) = \begin{array}{c} \triangle x \quad \triangle y \\ | \quad | \\ \text{---} U \text{---} \\ | \quad | \\ \triangle a \quad \equiv \\ \frac{1}{d_B} \end{array} \quad \begin{array}{c} \triangle x \quad \triangle y \\ | \quad | \\ \text{---} U \text{---} \\ | \quad | \\ \triangle a \quad \equiv \end{array} = P_{post}(a | xy, U)$$

$$P_{pre}(x | ab, U) = \begin{array}{c} \triangle x \quad \equiv \\ | \quad | \\ \text{---} U \text{---} \\ | \quad | \\ \triangle a \quad \triangle b \end{array} \quad \begin{array}{c} \triangle x \quad \equiv \\ | \quad | \\ \text{---} U \text{---} \\ | \quad | \\ \triangle a \quad \triangle b \end{array} \frac{1}{d_Y} = P_{post}(ab | x, U)$$

**Prior**  $P(a) = \frac{1}{d_A}$

**Data**  $P(x) = \sum_{i=1}^{d_A} \frac{1}{d_A} P_{pre}(x | a_i, \Phi) = \frac{1}{d_A} \text{tr} |x\rangle\langle x| \Phi[\mathbb{1}_A]$

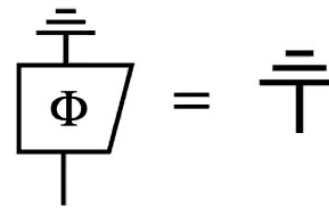
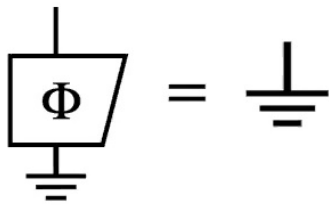
$$P_{post}(a | x, \Phi) = \frac{\text{tr} |x\rangle\langle x| \Phi[|a\rangle\langle a|]}{\text{tr} |x\rangle\langle x| \Phi[\mathbb{1}_A]} = \frac{P_{pre}(x | a, \Phi)}{\text{tr} |x\rangle\langle x| \Phi[\mathbb{1}_A]}$$

## Inference Symmetric Channels

$$P_{post}(a | x, \Phi) = \frac{P_{pre}(x | a, \Phi)}{\text{tr} |x\rangle\langle x| \Phi[\mathbb{1}_A]}$$

$$P_{post}(a | x, \Phi) = P_{pre}(x | a, \Phi)$$

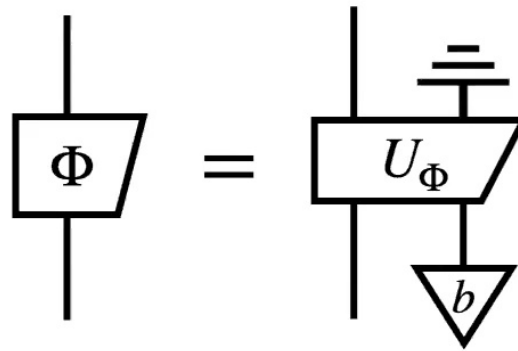
$$\iff \Phi[\mathbb{1}_A] = \mathbb{1}_X$$





## Purification

Any quantum channel can be understood in terms of a unitary interaction with an ancilla system.



This allows us to understand the inference asymmetry of the quantum channels.

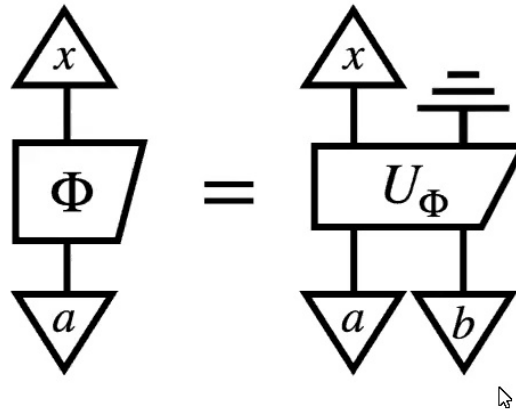
$$P_{post}(a | xb, U_\Phi) = \frac{P_{pre}(x | ab, U_\Phi)}{d_A P_{pre}(x | b, U_\Phi)}$$

$$P_{pre}(x | ab, U_\Phi) = P_{pre}(x | a, \Phi)$$

$$P_{pre}(x | b, U_\Phi) = \sum_{i=1}^{d_A} \frac{1}{d_A} P_{pre}(x | a_i b, U_\Phi) = \frac{1}{d_A} \sum_{i=1}^{d_A} P_{pre}(x | a_i, \Phi)$$

$$P_{post}(a | xb, U_\Phi) = \frac{P_{pre}(x | ab, U_\Phi)}{\text{tr} |x\rangle\langle x| \Phi[\mathbb{1}_A]} = P_{post}(a | x, \Phi)$$

$$P_{post}(x | a, \Phi) = P_{post}(a | xb, U_{\Phi})$$



The inference asymmetry of quantum channels is understood as an asymmetry in the boundary data.

*There exists a unique deterministic effect.*

**Mathematically correct:** the trace is the only CPTP map to the trivial space.

**Physically correct:** there is fundamental unpredictability in QM.

**But not a difference between past and future:** there is fundamental *unpost*dictability in QM.

## Maxim 2

An operation is a set  $E = \{E_x\}$  of CP maps such that  $\sum_x E_x$  is a CPTP map.

The probability of outcome  $x$  is given by  $P(x|\rho, E) = \text{tr } E_x[\rho]$ .

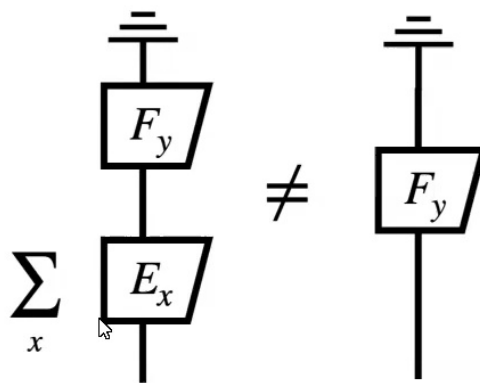
$$\sum_x \begin{array}{c} \text{---} \\ \text{---} \\ \square \\ E_x \\ \text{---} \end{array} = \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \end{array} \quad P(x|\rho, E) = \begin{array}{c} \text{---} \\ \text{---} \\ \square \\ E_x \\ \text{---} \\ \triangle \\ \rho \end{array}$$

If two operations  $\{E_x\}$  and  $\{F_y\}$  are composed in sequence:

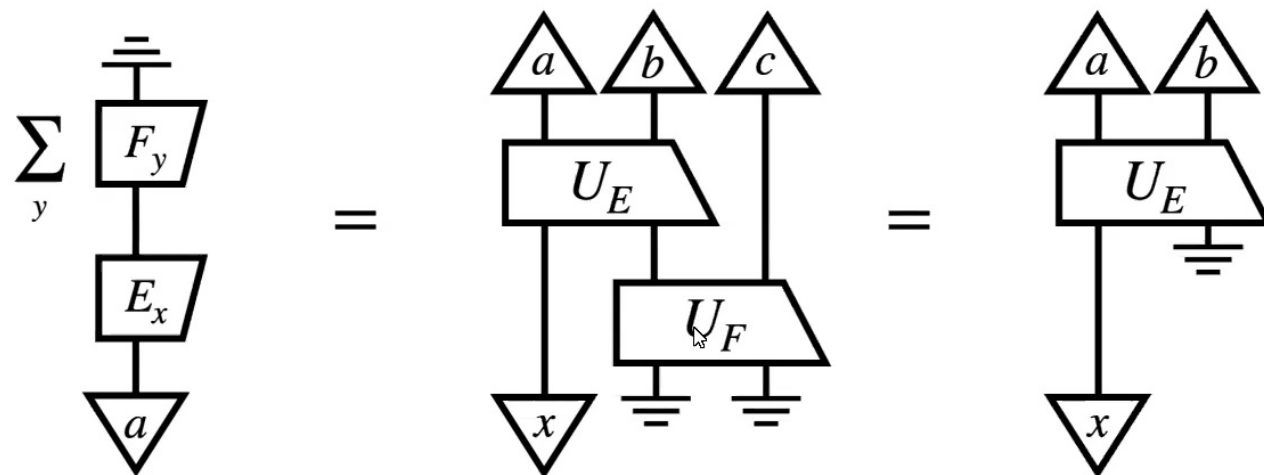
$$P(xy|\rho, F \circ E) = \text{tr } F_y[E_x[\rho]].$$

## Maxim 2

$$P(y|\rho, F \circ E) = \sum_x \text{tr} F_y[E_x[\rho]] = \text{tr} F_y[E[\rho]]$$

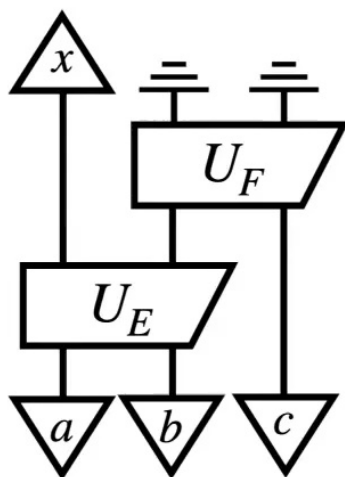


## Maxim 2

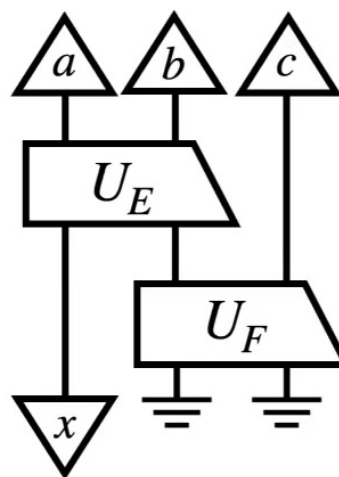


$$P(x | a, F \circ E) = P_{post}(x | abc, U_E^\dagger \circ U_F^\dagger)$$

## Maxim 2



↗





## Why the asymmetry?

There are two asymmetric aspects:

- We are interested in prediction
- We consider time-asymmetric boundary conditions

Both can be understood in terms of thermodynamics:

- We remember the past, and not the future
- We make choices that affect the future, not the past



Price, *Time's arrow & Archimedes' point*, Oxford University Press (1997)

Mlodinow and Brun, *Relation between the psychological and thermodynamic arrows of time*. Phys. Rev. E **89**, (2014)

Rovelli, *Agency in Physics*. arXiv:2007.05300 (2020)

Rovelli, *Memory and entropy*. arXiv:2003.06687 (2020)

Ismael, *How physics makes us free*, Oxford University Press (2016)

## Why the asymmetry?

Time-asymmetry comes in from the user of QM.

QI about correlations established between agents.

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Time-asymmetry comes in from the user of QM.

QI about correlations established between agents.

[Submitted on 12 Oct 2020]

<http://arxiv.org/abs/2010.05734>

# Quantum information and the arrow of time

Andrea Di Biagio, Pietro Donà, Carlo Rovelli

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## Take aways

Transition probabilities do not care about the direction of time.

There is a difference between known and unknown.

## Unscrambling

Diagrammatic calculus is useful for systemic thinking, very legible, and suited for "distributed" processes.

But... there is a strange mix between the "physical" and "inferential" aspect of the theory.



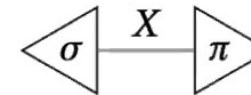
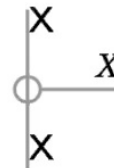
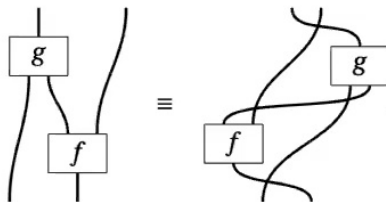
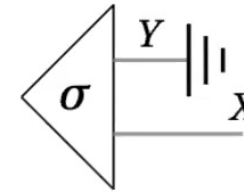
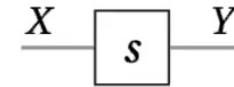
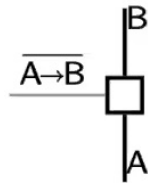
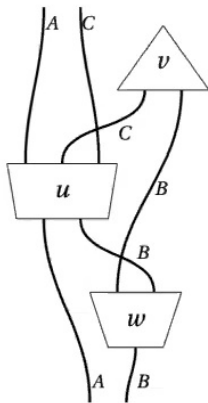
# Unscrambling

[Submitted on 7 Sep 2020 (v1), last revised 9 Sep 2020 (this version, v2)]

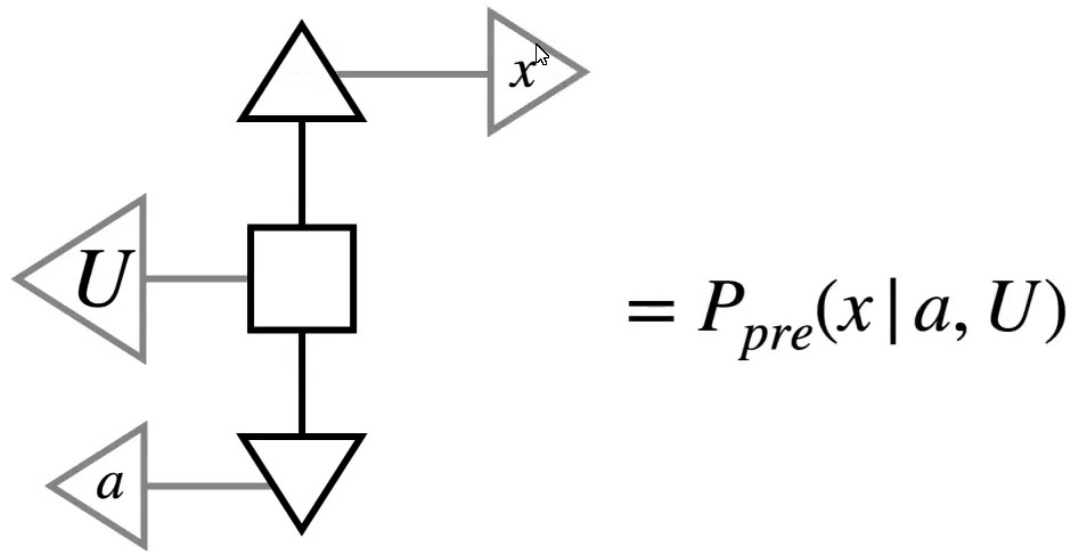
## Unscrambling the omelette of causation and inference: The framework of causal-inferential theories

David Schmid, John H. Selby, Robert W. Spekkens

$$\text{CAUS} \xrightarrow{e} \text{C-I} \xrightleftharpoons[\text{p}]{i} \text{INF}$$

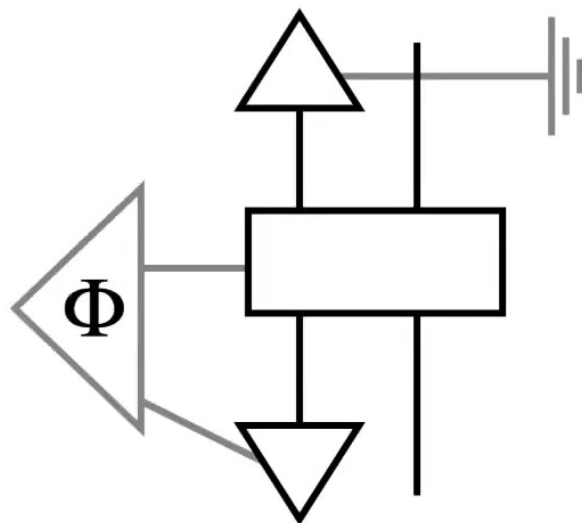


# Prediction





# Channel



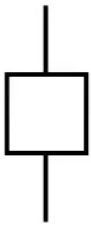
# Generators



Pure state preparation

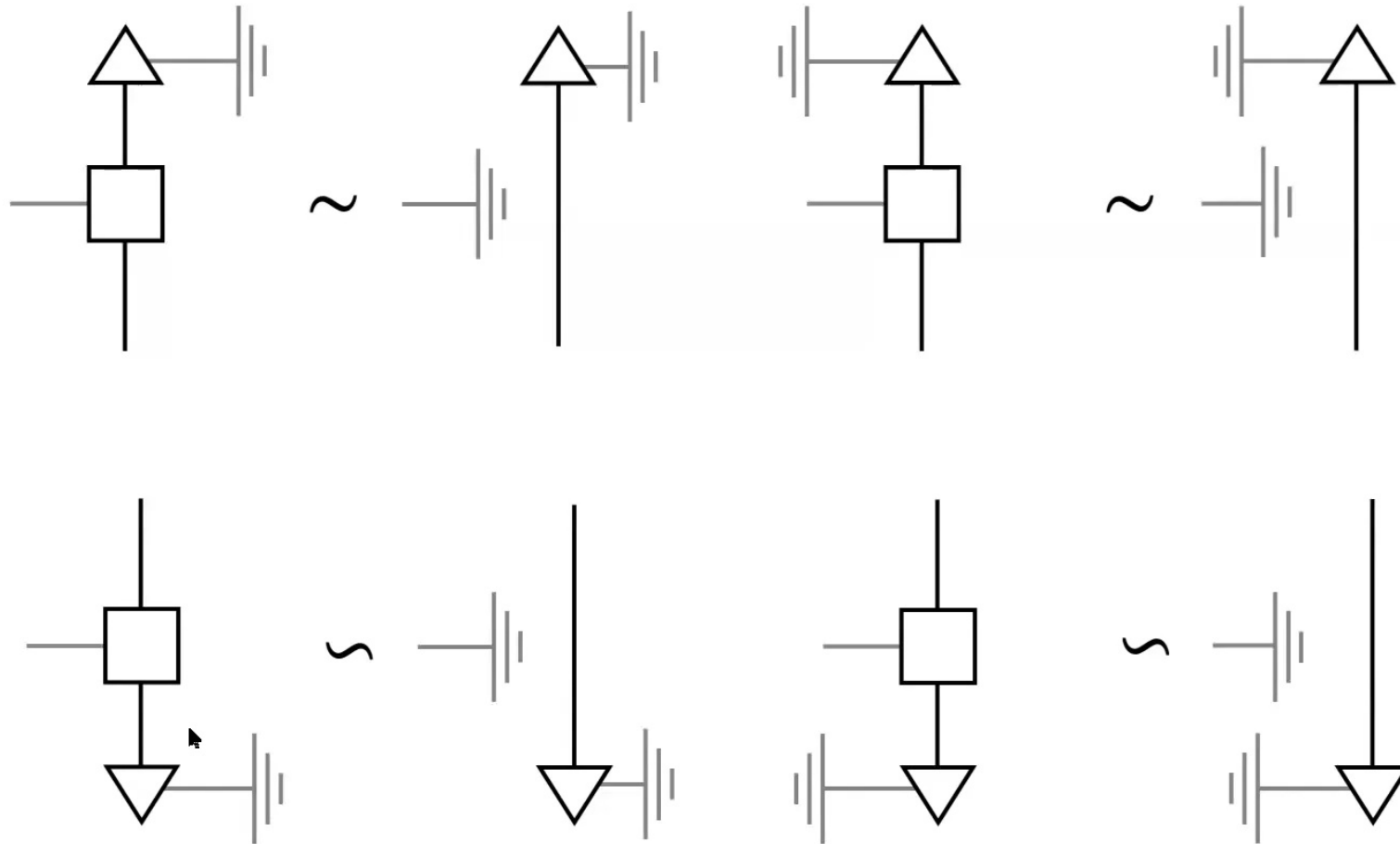


Pure state measurement

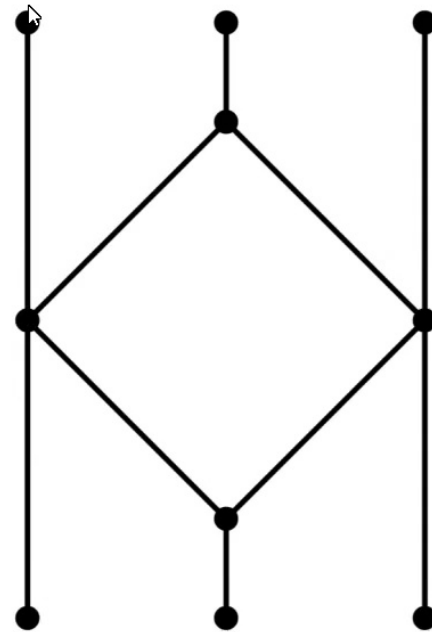
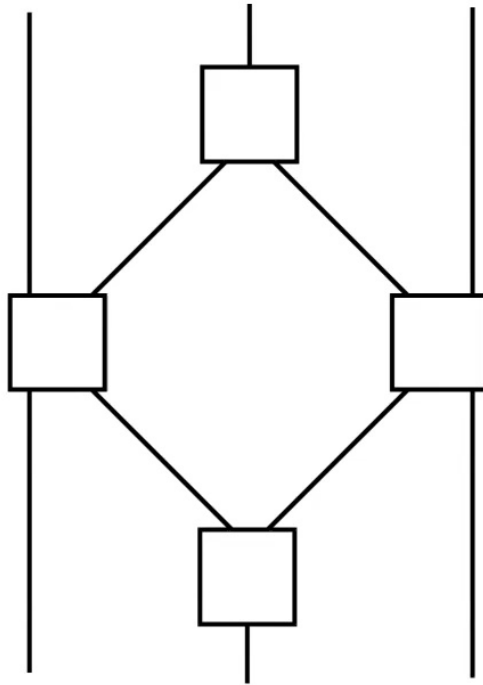


Unitary transformation

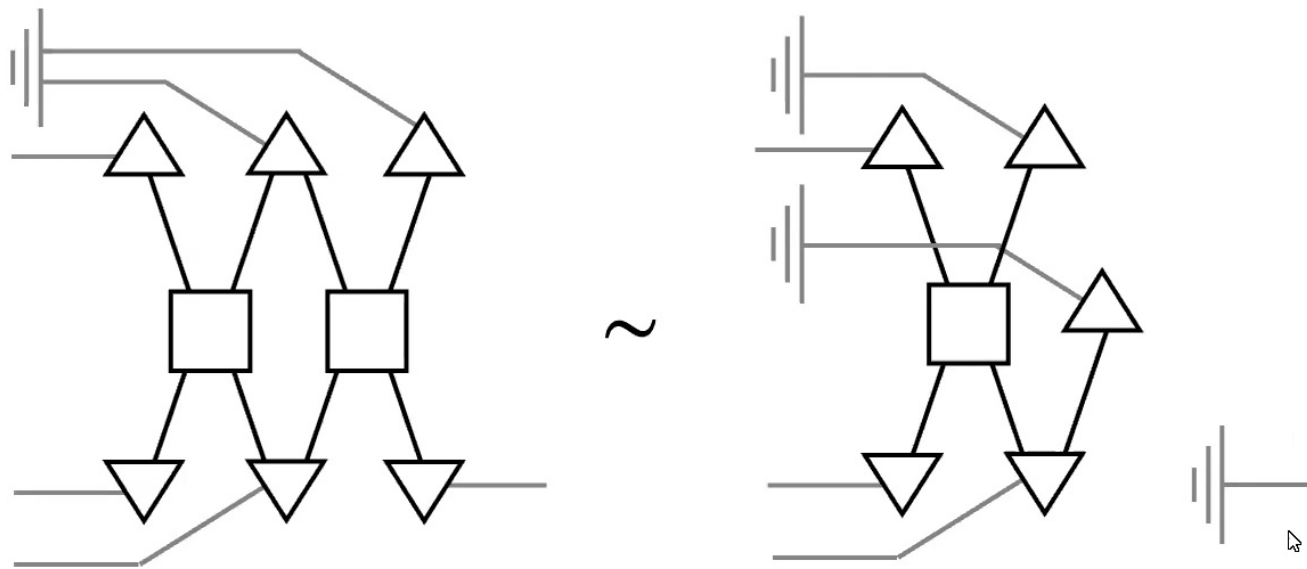
# Generators



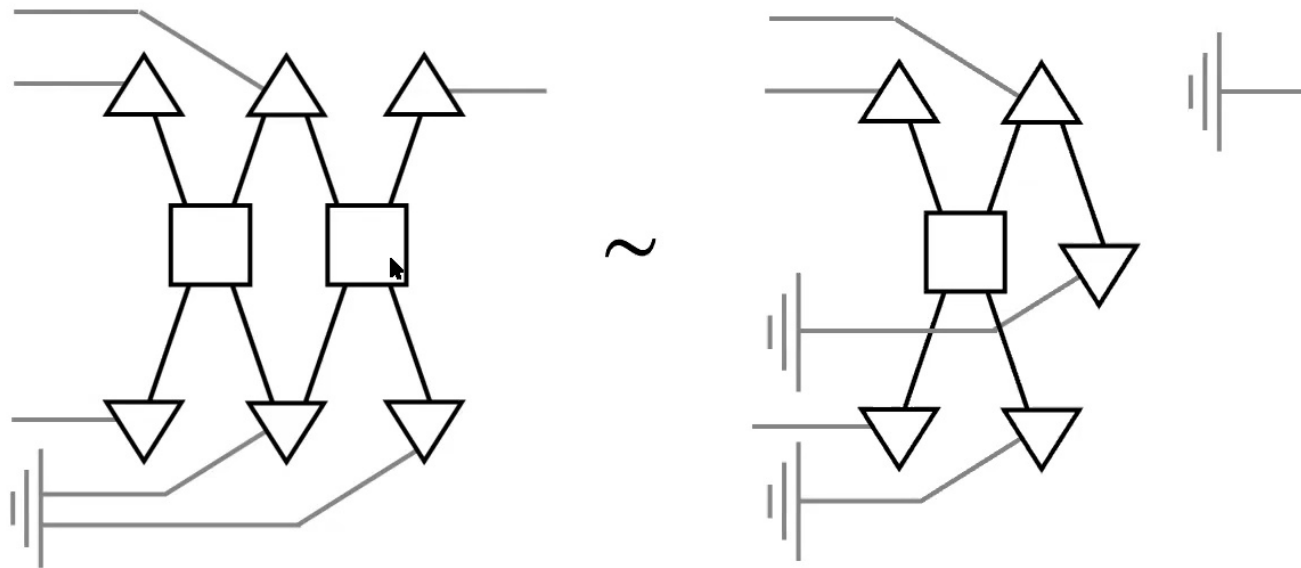
# Compositionality



# Signalling



# Signalling



Separate causation (necessary correlations) from inference.

Think about causation time-symmetrically.

### **More goals:**

Use Gibbs-preserving maps to talk about thermodynamical aspects.

Use a more elaborate quantum/classical interface.

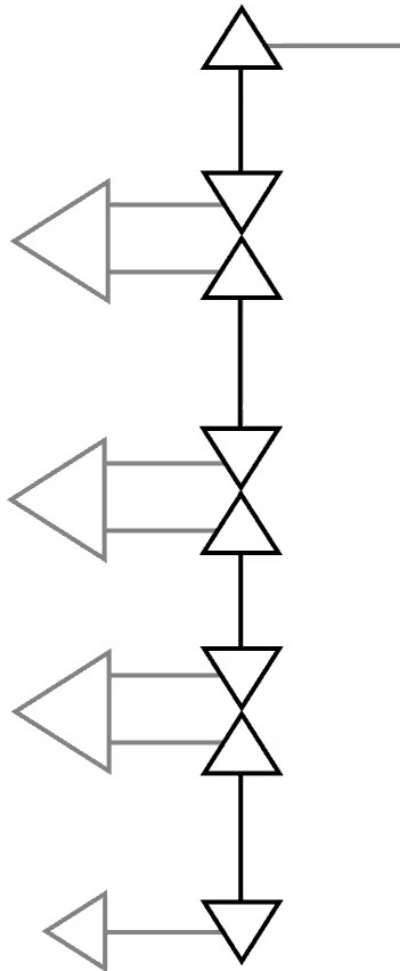
# Time-symmetric reconstruction?

Toolbox for reconstructing quantum theory from rules on information acquisition

Philipp Andres Höhn,  
Quantum 1, 38 (2017).

Quantum theory from questions

Philipp Andres Höhn and Christopher S. P. Wever  
Phys. Rev. A **95**, 012102 – Published 3 January 2017



Replace "questions and answers" with  
"interactions and values of observables".

QM allows to calculate the probability of  
an event, given other events.



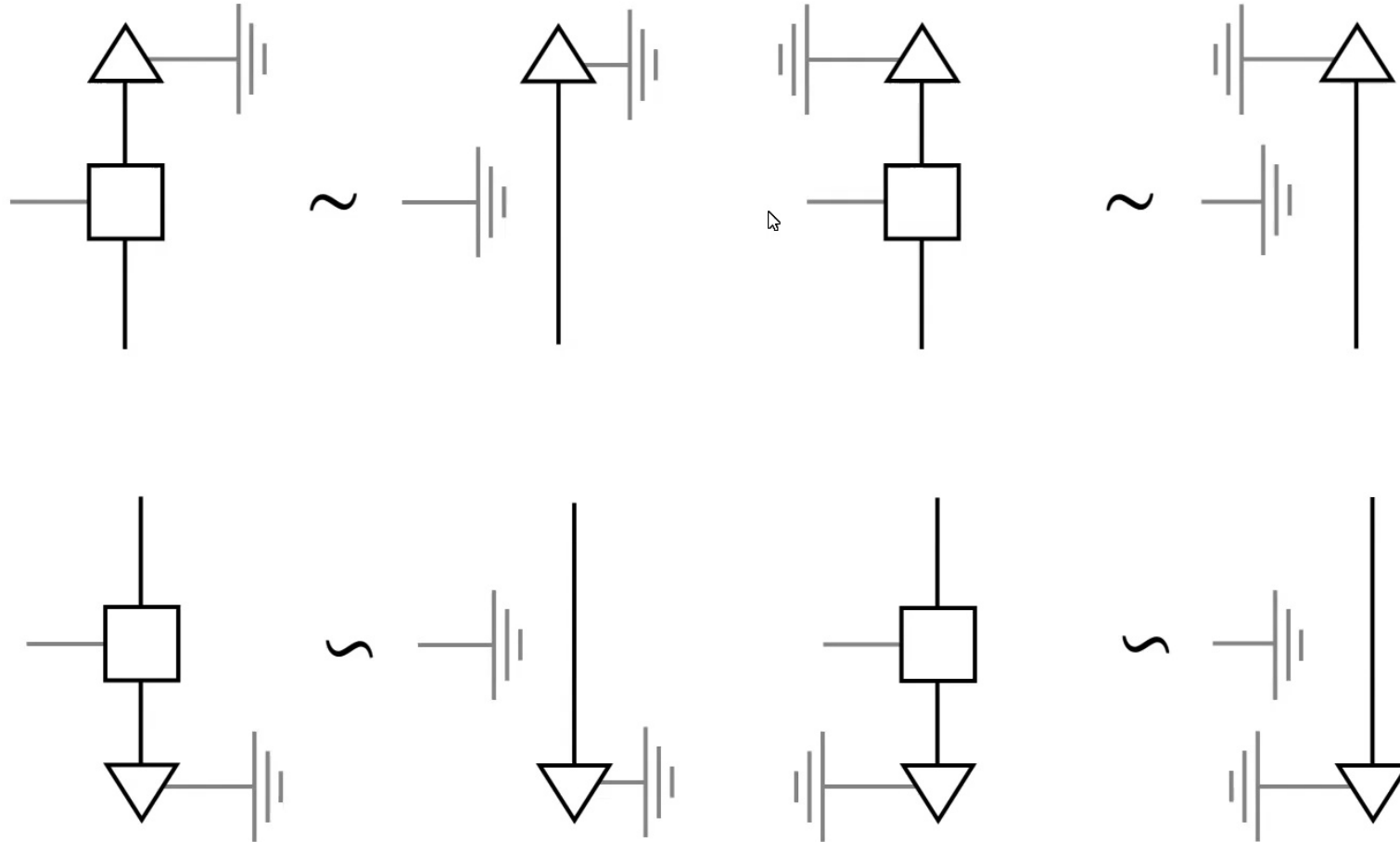
Thank you

To be continued....

Probability, like time, is a concept invented by humans, and humans have to bear the responsibility for the obscurities that attend it.

Wheeler, *Information, Physics, Quantum*, in Complexity, Entropy and the Physics of Information (1990)

# Generators



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Think about causation time-symmetrically.

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Thank you

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## Maxim 2

